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PRECISION OF THE LONG BASELINE ACOUSTIC
NAVIGATION SYSTEM USED BY PEGASUS

by

Margaret F. Haskell

June 1991

Thesis Co-Advisors:

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Kurt J. Schnebele

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Precision of the Long Baseline Acoustic
Navigation System Used by Pegasus

by

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ABSTRACT

A least squares algorithm is developed to solve for the trajectory and transponder array coordinates of the current velocity profiler, Pegasus. Measurement residuals and parameter precision are computed for data quality analysis. Travel times from a maximum of four seafloor transponders, pressure sensor depths, and transponder positions are input with their respective accuracy estimates. The algorithm is used to analyze a 2250 m profile from the Monterey Canyon with four transponders, one of which had not been positioned. This transponder's unknown position is found and problems in the other array coordinates identified. Transponder coordinate precision improves by factors of ten in the horizontal and five in depth, to about 13 m (Drms) and 2 m (1 σ) respectively. Trajectory precision is about 7 m (Drms) horizontal, with high correlation between points. Thus, the precision of horizontal velocity components, determined by time differencing points, is better than 11 cm/s (1 σ). Depth precision is better than 3 m (1 σ), except in the deeper portions where anomalous pressure residuals near the depth of the transponder array suggest systematic pressure errors needing further study.

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I. INTRODUCTION

The Oceanography Department of the Naval Postgraduate School is presently conducting a study of ocean current velocities in the waters off the central California coast in the area of the Monterey Canyon. During the course of the study, several stations have been established for the collection of data. Station 10, northernmost in this chain of stations and the primary focus for analyses presented in this thesis, contains four seafloor transponders, one of which was considered to have an essentially unknown position at the beginning of the work described here.

The main oceanographic instrument used in this survey is the dropsonde Pegasus. As it descends and ascends through the water column, it records at 16 second intervals, both the data from oceanographic sensors and the time for return travel of acoustic signals from transponders previously set in place on the ocean floor.

In the past, Pegasus-generated velocity studies have used the raw data in various ways. The trajectory of the instrument has been fitted to a curve which was then differentiated with respect to time in order to produce the horizontal velocity components (Halkin et al., 1985). Alternatively, and as is the present practice at NPS, velocities have been

calculated from position differences and then smoothed with a spatial filter such as a seven point running average.

This study outlines a method of determining the current velocities and their precision by applying a least squares adjustment procedure, adapted from geodetic survey techniques, whereby all observational data are used simultaneously. Specifically, three basic problems will be analyzed:

- * The precision of the transponder coordinates from which the position of Pegasus is derived.
- * The precision of the Pegasus positions and consequently the current velocities derived from Pegasus navigation.
- * The depth at which pressure becomes critical in solving for velocities.

Discussions of this topic will appear in the following sequence:

- * Chapter II provides background information on the transponder network and the Pegasus system and its positioning.
- * Chapter III discusses sources of error resulting from systematic and observational inaccuracies in the data.
- * Chapter IV gives a brief explanation of the least squares adjustment process.
- * Chapter V describes the final procedure used.
- * Chapter VI discusses the results obtained.
- * Chapter VII provides conclusions and recommendations for future consideration.
- * Appendices contain algorithms and explanations of the Fortran programs used.

II. BACKGROUND

The area being surveyed is composed of ten stations located just off the California coast approximately between $36^{\circ}05'$ and $36^{\circ}39'$ North Latitude, and $122^{\circ}08'$ and $124^{\circ}13'$ West Longitude. Of particular interest here is Station C10, northernmost in the chain and located in the Monterey Canyon about ten miles west of Monterey (see Figure 1).

A. TRANSPONDER NETWORK

At this station four transponders were deployed, horizontally separated by distances somewhat equivalent to transponder depths (around 2000 m). Three of these transponders, responding at 11.5 kHz, 12.0 kHz, and 12.5 kHz, respectively, were located by the survey vessel according to traditional methods as described below. However, the bandwidth of the shipboard receiving instrument was too narrow to pick up the 13.5 kHz signal from the fourth transponder, and therefore its position was largely unknown. (Table 1 defines transponder abbreviations, their frequencies, and their approximate depths. Figure 2 shows the survey net for Station C10.)

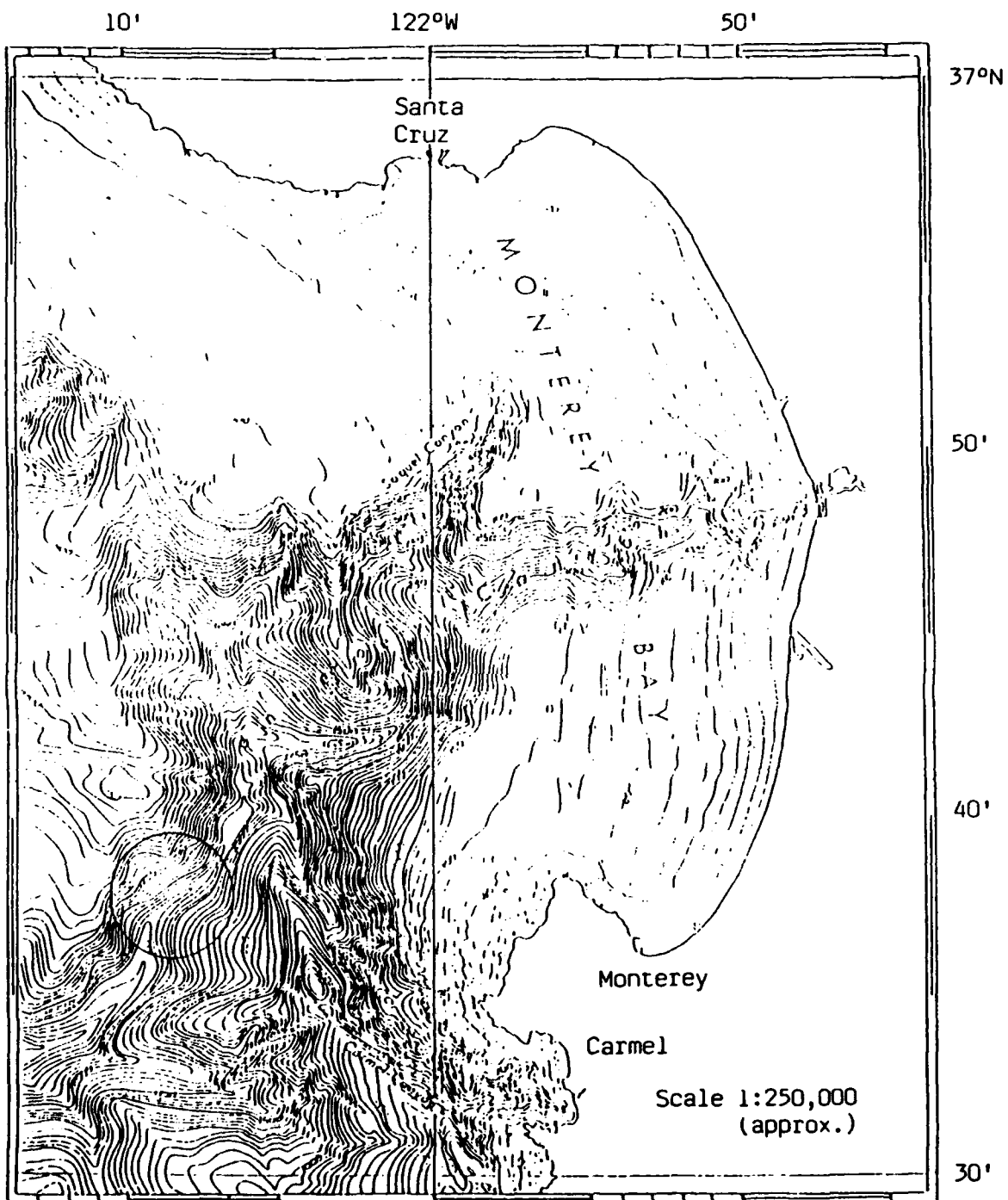


Figure 1. General Area of Station C10

TABLE 1
C10 TRANSPONDERS

Transponder 1	T1	12.5 kHz	1880 m
Transponder 2	T2	12.0 kHz	1990 m
Transponder 3	T3	11.5 kHz	2230 m
Transponder 4	T4	13.5 kHz	1890 m

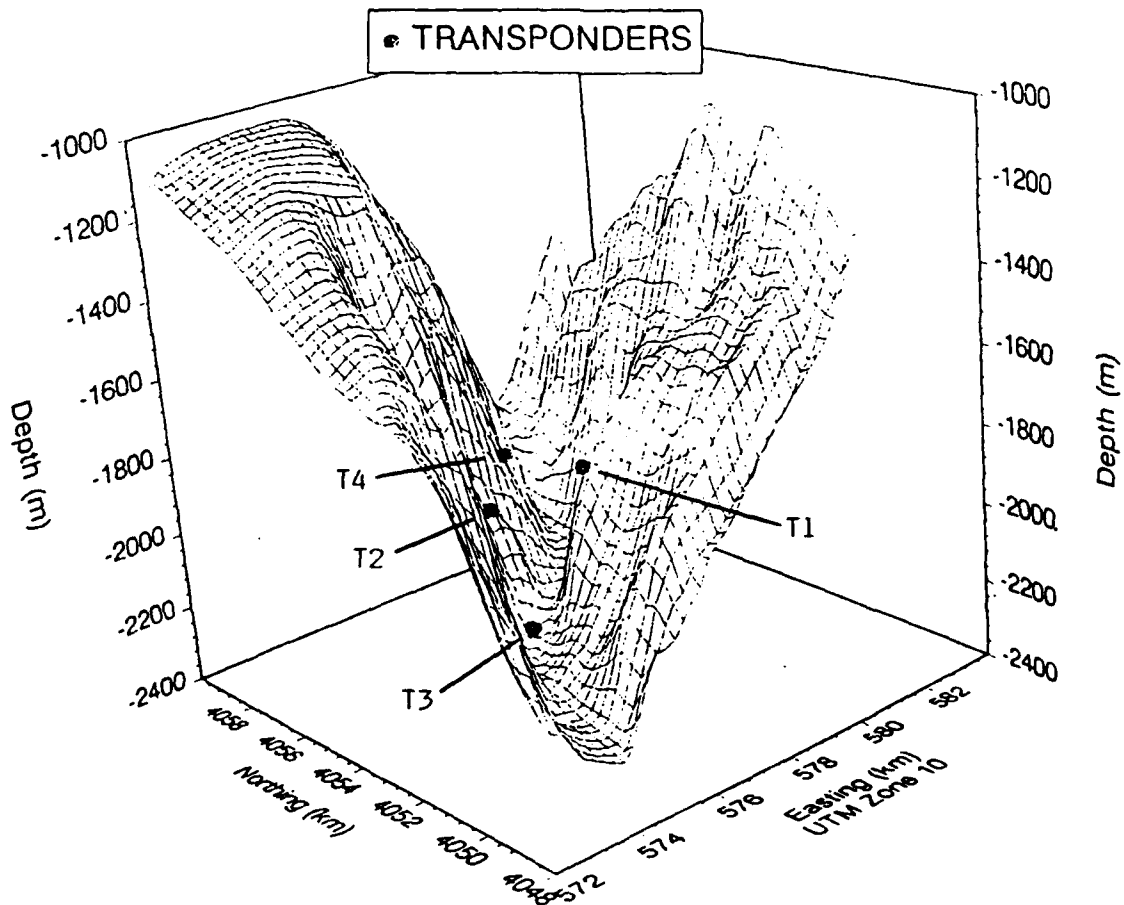


Figure 2. Station C10 Survey Net

B. SURVEYING A TRANSPONDER NETWORK

1. Transponder Depth

To determine each available transponder depth, the survey ship homes in on the instrument's signal until a minimum time is recorded, thereby assuming the ship to be directly over the instrument (see Figure 3). An appropriate harmonic mean sound velocity multiplied by one way signal travel time produces the depth of the transponder.

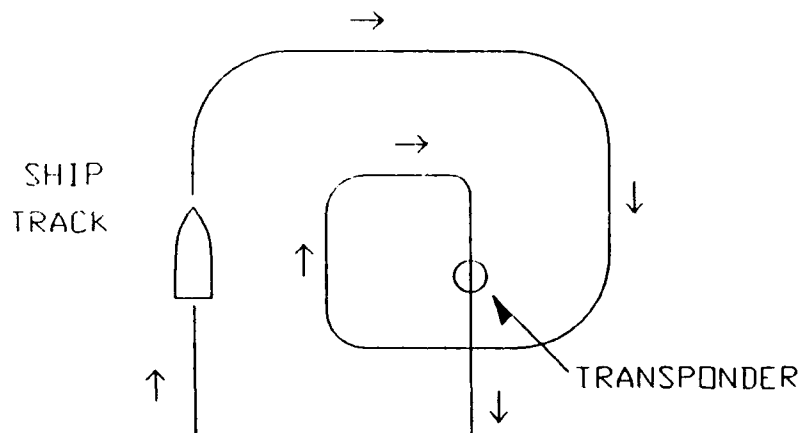


Figure 3. Determining Transponder Depth

2. Baselines

To establish the relative horizontal x and y coordinates of transponders, baselines are determined between the instruments. Running at a relatively slow speed (say 4-6 kts), the survey vessel attempts to cross each baseline at a

90° angle. The process is repeated in the opposite direction. This procedure is then duplicated, several more times if possible. Careful observation of the analog trace that records round-trip travel times of the acoustic signal between the survey vessel and the two transponders indicates when the shortest distance has been reached, as shown in Figure 4.

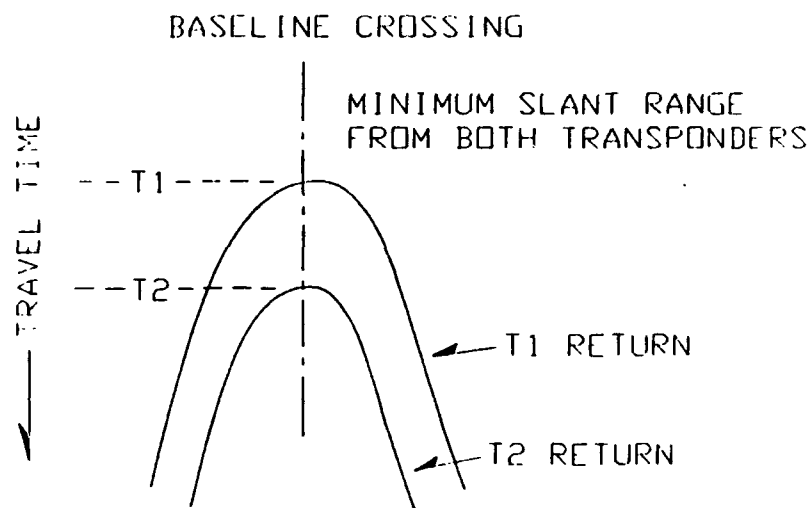


Figure 4. Analog Trace of a Baseline Crossing

Signals show up on the trace as parabolas approaching one another. They will lie vertically in line if the baseline is crossed at a 90° angle and therefore the shortest distance to each of them is reached simultaneously. At this point the ship will lie in a vertical plane containing the two transponders. The sum of the two horizontal ranges will be

at a minimum and therefore establish the baseline (see Figure 5).

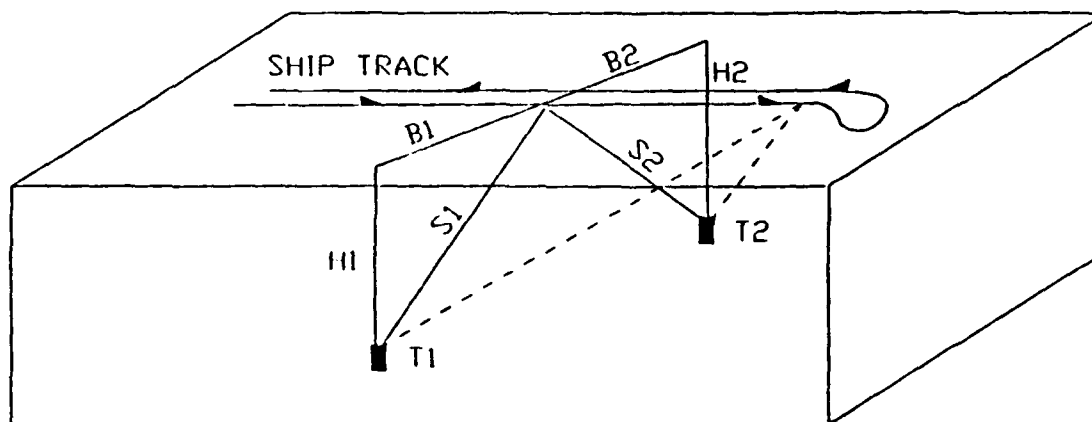


Figure 5. Baseline Determination

The time of crossing, ship's course, latitude/longitude, and Loran coordinates are noted. Time units for each minimum range are measured and then multiplied by the appropriate harmonic mean sound velocity to obtain slope distances. These distances, the depths of the two transponders, and the ship's course are then used to calculate the horizontal baseline length and the azimuth between the two transponders. Finally, a local xy plane coordinate system is established, setting the position of one of the transponders at 0,0, and positioning the other transponder relative to it.

Many factors combine to make the information on this analog trace subjective and inaccurate. The analog paper may

be set to progress at different rates producing sweeps of 0.5 s, 1 s, or 2 s, etc. The trace may actually "wrap around" one or more times as the time interval gets longer and longer, making page integer resolution difficult. At a one second sweep, the full page represents (after conversion from time to distance) meters traveled in one second, i.e., 1500 m/s.

Furthermore, if the baseline is not crossed perpendicularly, a time offset between minimum ranges is observed which decreases the length of the baseline. This error can be removed to some extent by multiplying the time offset by the ship's speed and then by geometric relationships, computing the correction. This situation is illustrated in Figure 6 where T1-T2 represents the actual baseline, A, B represent respective points on the analog trace where minimum slant ranges are indicated, and a, b represent respective minimum slant ranges to each transponder.

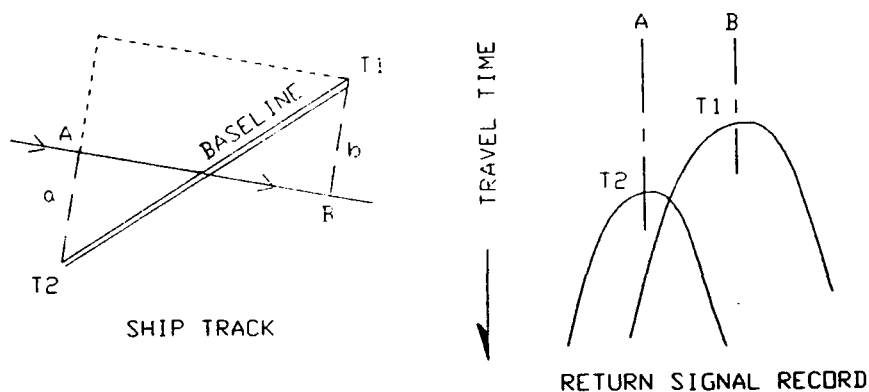


Figure 6. Time Offset

In addition to the above possibilities for error, an inaccurate course heading at the time of baseline crossing will alter the geometry of the array, resulting in inaccurate coordinates. Further discussions of transponder depth and baseline determinations can be found in Kuo (1985), McKeown (1975), and Hart (1967). For an assessment of transponder network accuracy, refer to Chapter III.

Figure 7 illustrates the C10 transponder network and the approximate position of Pegasus drop #157 positioned in a local coordinate system using T1 (12.5 kHz) as 0,0.

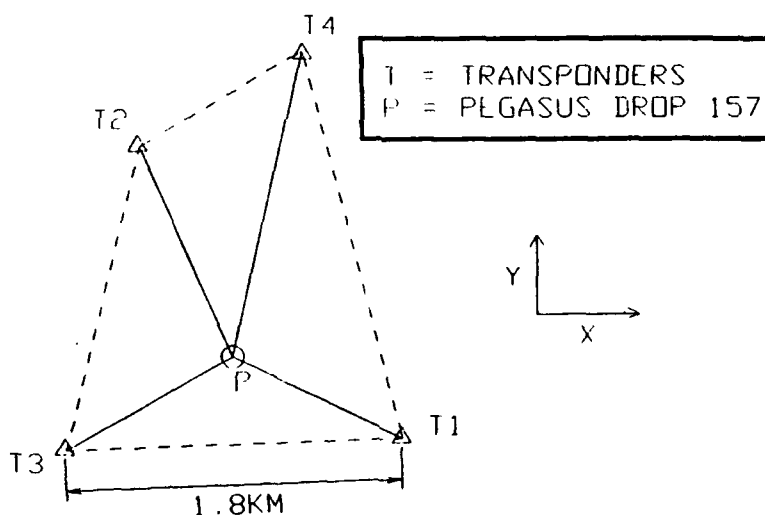


Figure 7. The C10 Local Coordinate System

C. DESCRIPTION OF THE PEGASUS SYSTEM

The oceanographic instrument Pegasus is a free-falling, acoustically-tracked velocity profiler. It was developed by H.T. Rossby and D. Dorson at the University of Rhode Island a decade ago to fill a need for an instrument which would accurately record the fine scale vertical structure in the ocean and prove inexpensive and easy to handle at sea. It provides a means to measure absolute velocity components throughout the water column. (For the seminal paper on Pegasus, see Spain et al., 1981.) Prior uses of Pegasus in studies of the Gulf Stream off the east coast of the U.S. have been documented in a paper and data report by Halkin et al. (1985).

The position of the instrument as it descends and ascends is determined by travel time of signals emitted by Pegasus at 10 kHz every 16 seconds which are heard and then answered at other frequencies by transponders previously set in place on the ocean floor, and by pressure readings. These time intervals, and temperature, conductivity, and pressure data from oceanographic sensors, are recorded and stored in a microprocessor-controlled memory in the instrument to be later down-loaded aboard ship when the Pegasus is recovered.

The Pegasus model currently adapted for use by the Naval Postgraduate School is manufactured by Benthos. It is housed in a 17-inch glass sphere protected by a hard hat and, according to manufacturer claims (Benthos, 1989), provides operation to full ocean depth, integral flotation, no

corrosion, and large battery capacity allowing for 100 deployments without opening. (A normal NPS school cruise uses about 20 deployments.) It weighs approximately 45 kg in air and carries eight kg of expendable weight which sink the profiler in this present study at about 27 m/min. Six receiver channels for communication with transponders are available with frequencies located at 0.5 kHz intervals ranging from 11.5 kHz to 14.0 kHz. NPS supplied SEA BIRD model SBE-3 temperature sensor and SEA BIRD model SBE-4 conductivity sensor are externally mounted with electrical interfaces, along with a Paroscientific model 410KT pressure transducer with a companion Intelligent Transmitter circuit board.

D. PEGASUS POSITIONING

1. Computation of Pegasus Coordinates

The position of Pegasus at each 16 second interval can be determined by solving a system of equations using the formula:

$$\text{Time} = \text{Distance} / \text{Sound Velocity}$$

Using one-way signal times from two transponders (see Figure 8), the depth of Pegasus as derived from pressure information, and an appropriate sound velocity, there will then be two equations in two unknowns, allowing for a solution

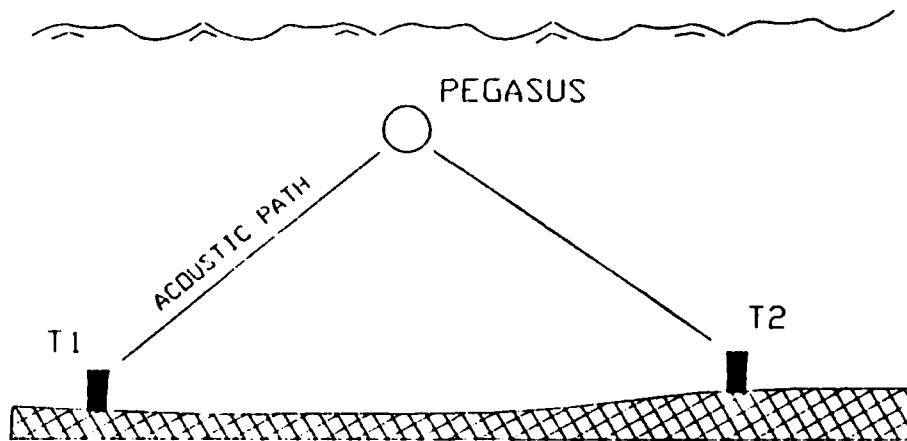


Figure 8. Determining Pegasus Position

of XP and YP at a given Pegasus position. The solution is derived from the equations:

$$\text{Time}_1 = [(\text{XP} - \text{X}_1)^2 + (\text{YP} - \text{Y}_1)^2 + (\text{ZP} - \text{Z}_1)^2]^{1/2}/V$$

$$\text{Time}_2 = [(\text{XP} - \text{X}_2)^2 + (\text{YP} - \text{Y}_2)^2 + (\text{ZP} - \text{Z}_2)^2]^{1/2}/V$$

where:

- Time_1 = one-way travel time from transponders 1,2;
- V = effective sound velocity along the path between Pegasus and the transponder;
- $\text{XP}, \text{YP}, \text{ZP}$ = Pegasus position coordinates (ZP is assumed known from the simultaneous pressure observations);

X_i, Y_i, Z_i = transponder coordinates (assumed known).

The traditional oceanographic method outlined above calculates Pegasus position coordinates. However, it lacks a means of obtaining precision estimates for the derived positions, and it is not able to use all the observed data simultaneously. Furthermore, there is no redundancy in the computational process and thus systematic errors or blunders can escape unnoticed.

2. Estimating Horizontal Current Velocity

Current velocity vectors u and v (describing velocity in the x and y directions, respectively) between two Pegasus positions, can be obtained by taking the difference between coordinates and dividing by the time interval:

$$u = (XP_2 - XP_1)/dt$$

$$v = (YP_2 - YP_1)/dt$$

$$dt = 16 \text{ s}$$

This method, like the one mentioned in the previous section, does not provide estimates of precision and does not make use of all available data.

III. SOURCES OF ERROR

Davis et al. (1981, pp. 15-20) defines measurement error as the difference between a measured and a "true" value and describes these errors as being generally of three types:

- * blunders or mistakes.
- * systematic errors.
- * random errors.

Blunders can be caused by carelessness, equipment failure, or false interpretation. They are usually large enough to be easily spotted when results are analyzed.

Systematic errors follow a defined pattern or system, which when discovered, can usually be expressed mathematically. Such factors as observer limitations, instrument imperfections or inadequate calibration, meteorological conditions, or poor choice of mathematical model can produce the pattern which will remain consistent as long as the elements of the system remain the same. Systematic errors are not eliminated by repetition of measurements. They must be ferreted out and either removed from the observations or their effects added to the model. Thus, to begin such an analysis, serious consideration has to be given to locating and defining errors resulting from systematic and observational inaccuracies in the data.

Random errors are those that remain after mistakes and systematic errors have been accounted for. The values of these errors should be unbiased and will ideally distribute themselves about a mean of zero. It is these random errors that the least squares process attempts to minimize. In the analysis described in this thesis, a considerable amount of time was spent in determining what the estimates of the *a priori* standard deviations of these random errors should be, both from the point of view of reasonable physical reality and meaningful results. The adjustment technique uses these *a priori* estimates to weight the contribution of each observation to the final solution.

The following potential error sources were studied and, where applicable, appropriate standard deviations of the errors were then entered as weights into the least squares adjustment model:

- * Transponder coordinates.
- * Signal time.
- * Velocity of sound.
- * Pressure/Depth relationship.

A. TRANSPONDER SURVEY

Of prime importance is the precision of the transponder positions. Determining these coordinates is a difficult and time-consuming process. As noted before, in the C10 transponder net, the location of the 4th transponder could not

be determined by traditional methods because the band width of the ship's receiver was not wide enough to pick up the 13.5 kHz signal. Also, it was not possible at the time to determine more than two of the three baselines between the three "known" points, thereby precluding a mathematical closure of the triangle.

1. Transponder Depth

To establish a reasonable estimate for the standard deviation of transponder depth error, it was noted that any horizontal offset resulting from the ship's not being directly overhead always produces a positive depth error, i.e., the slant range must be longer than the vertical. Figure 9 is similar to Spain et al. (1981, p. 1557) and illustrates this process.

The solution is as follows:

$$(H + h)^2 = x^2 + H^2$$

$$2Hh + h^2 = x^2$$

$$\frac{x^2}{2H} = h + \frac{h^2}{2H} = h(1 + \frac{h}{2H})$$

where:

H = true transponder depth;

h = depth error.

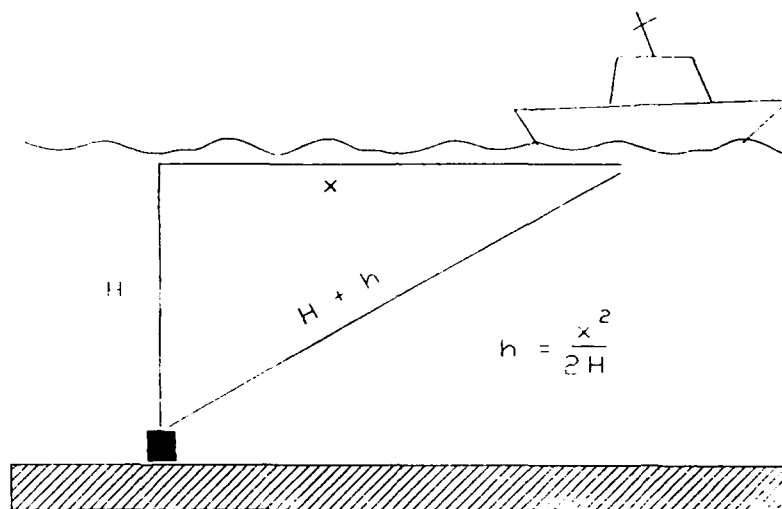


Figure 9. Transponder Depth Error

If $h \ll H$, the last term can be disregarded, and

$$h \approx \frac{x^2}{2H}$$

A realistic estimate of the ability of a survey vessel to cruise directly over a transponder by using this method is about 100 meters (Schnebele, 1990a). Given the uneven canyon terrain of this survey which makes resolving the position of the shoalest depth even more difficult, it seemed reasonable to ascribe an error of 200 m to the offset. Thus, if the true depth lies at 2000 m and there is a horizontal offset of 200 m, there will be a depth error of about 10 m.

Therefore, a $\sigma_z = 10$ meters was chosen for transponders 1, 2, and 3. Since there was no observed depth for Transponder 4, a depth of 1900 meters was read from the latest NOAA chart using the position of transponder drop¹ and, since it had not been surveyed in, a larger standard deviation of 100 meters attached to it.²

$$\sigma_{(T1z, T2z, T3z)} = 10 \text{ m}$$

$$\sigma_{T4z} = 100 \text{ m}$$

2. Baselines

In determining baselines, three sources of error predominate:

- * Minimum range reading taken when the ship is offset from the actual baseline.
- * Transponder depth error propagating into the computed baseline length.
- * Error in ship's course affecting baseline azimuth.

In a simplified ideal situation where the baseline is crossed at a 90° angle in the center of the line and the depths of the two transponders are the same, the expected

¹The transponder position was calculated from Loran signals which are based on North American Datum 27. The NOAA chart uses NAD83. Positions may vary up to 100 meters between the two datums in this locality.

²After the October 17, 1989 earthquake, a depth measurement of 1931 meters was made of the 4th transponder by NPS scientist Tarry Rago in a submersible.

error in baseline length from measured slant ranges can be calculated as follows (see Figures 10 and 11).

a. Baseline Error Due to R

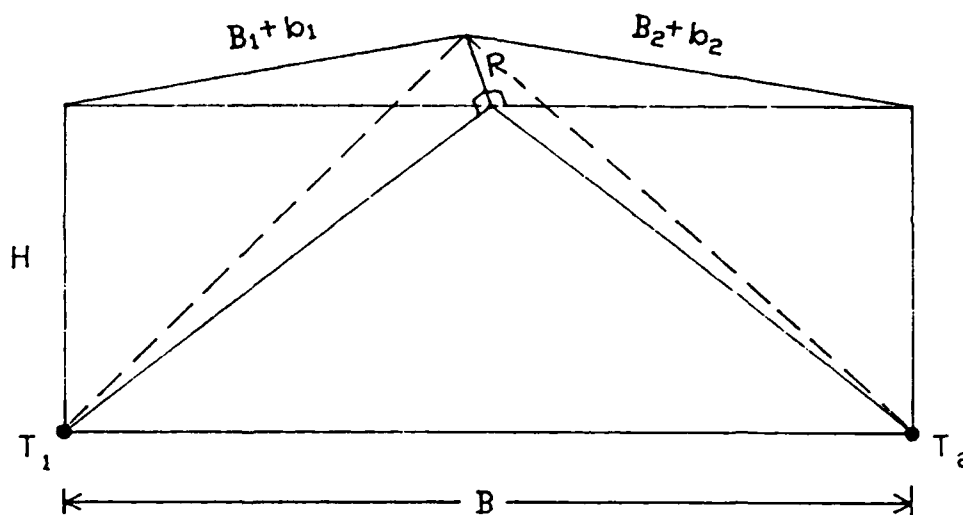


Figure 10. Baseline Error

- R = distance off baseline when slant ranges were measured;
- B = true baseline between transponders ($B = B_1 + B_2$);
- b = baseline error ($b = b_1 + b_2$);

$$(B_1 + b_1)^2 = R^2 + B_1^2$$

$$2B_1b_1 + b_1^2 = R^2$$

$$b_1(2B_1 + b_1) = R^2$$

$$b_1 = \frac{R^2}{2B_1 + b_1} = \frac{R^2}{2B_1(1 + \frac{b_1}{2B_1})} \rightarrow \frac{R^2}{2B_1} (1 - \frac{b_1}{2B_1} + (\frac{b_1}{2B_1})^2 - \dots)$$

Since $(\frac{-b_1}{2B_1} + \dots) \ll 1$,

$$b_1 \simeq \frac{R^2}{2B_1} \text{ and, similarly, } b_2 \simeq \frac{R^2}{2B_2}$$

If, as stated above, $B_1 = B_2 = B/2$, then $b_1 = b_2 = b/2$.

$$b_1 + b_2 \simeq \frac{R^2}{2B_1} + \frac{R^2}{2B_2} \simeq \frac{R^2}{2} (\frac{1}{B/2} + \frac{1}{B/2}) \simeq \frac{R^2}{2} (\frac{4}{B})$$

$$b_R \simeq \frac{2R^2}{B}$$

This error increases the length of the baseline.

b. Baseline Error Due to h

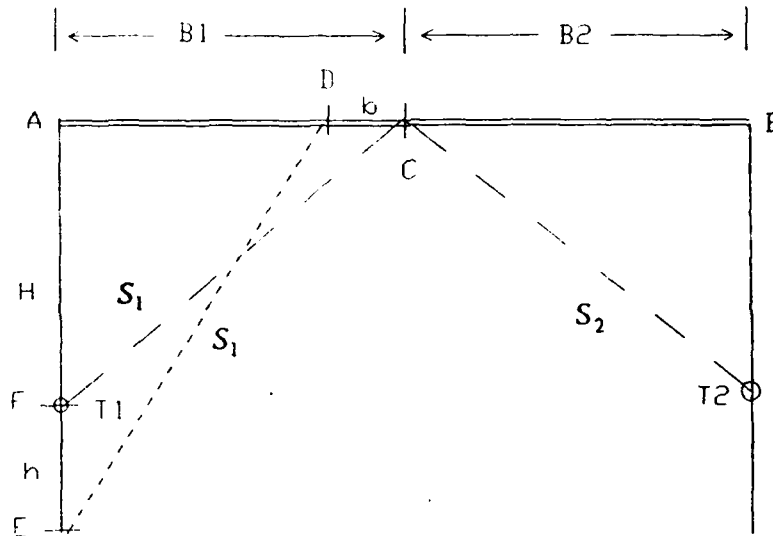


Figure 11. Relationship of Baseline and Depth Errors

H = true transponder depth;

h = depth error;

S_1 = slant range from ship to transponders 1,2.

$$S_1^2 = H_1^2 + B_1^2 \quad (1)$$

$$S_1^2 = (H_1 + h_1)^2 + (B_1 + b_1)^2 \quad (2)$$

S_1 remains constant.

Eq. (2) - Eq. (1):

$$0 = 2H_1h_1 + h_1^2 + 2B_1b_1 + b_1^2$$

$$2B_1b_1 \left(1 + \frac{b_1}{2B_1}\right) = -2H_1h_1 \left(1 + \frac{h_1}{2H_1}\right)$$

$$\frac{B_1b_1}{\left(1 + \frac{h_1}{2H_1}\right)} = - \frac{H_1h_1}{\left(1 + \frac{b_1}{2B_1}\right)}$$

But:

$$\frac{B_1b_1}{\left(1 + \frac{h_1}{2H_1}\right)} = B_1b_1 \left(1 - \frac{h_1}{2H_1} + \left(\frac{h_1}{2H_1}\right)^2 - \dots\right)$$

$$\frac{-H_1h_1}{\left(1 + \frac{b_1}{2B_1}\right)} = -H_1h_1 \left(1 - \frac{b_1}{2B_1} + \left(\frac{b_1}{2B_1}\right)^2 - \dots\right)$$

Since $\left(\frac{b_1}{2B_1} + \dots\right) \ll 1$, and $\left(\frac{h_1}{2H_1} + \dots\right) \ll 1$, then

$$B_1b_1 \simeq -H_1h_1$$

$$b_1 \simeq - \frac{H_1h_1}{B_1} .$$

If $H_1 = H_2 = H$, $h_1 = h_2 = h$, $B_1 = B_2 = B/2$, $b_1 = b_2 = b/2$, then

$$b_h = b_1 + b_2 \simeq -Hh \left(\frac{1}{B_1} + \frac{1}{B_2}\right) \simeq -Hh \left(\frac{1}{B/2} + \frac{1}{B/2}\right)$$

$$b_h \simeq - \frac{4Hh}{B} .$$

As shown in Figure 11, the hypotenuse representing the given slant range of the signal must be the same in both triangles ADE and ACF. FC must equal ED. Therefore, if the assumed depth is increased by error, then the computed baseline must consequently decrease.

c. Total Baseline Error

Thus the total baseline error $b_R + b_h$ becomes:

$$b_{Total} = b_R + b_h \simeq \frac{1}{B}(2R^2 - 4Hh)$$

Using typical figures of: $B = 2000$ m, $H = 2000$ m, $h = 10$ m, and $R = 100$ m, it should be possible to obtain a value for $b_{Total} = \pm 30$ m.

If the transponder depths are not the same, and the baseline is not crossed right at the center but is crossed on a perpendicular heading, the baseline error will be:

$$b_{Total} \simeq \frac{R^2}{2} \left(\frac{1}{B_1} + \frac{1}{B_2} \right) - \left(\frac{H_1 h_1}{B_1} + \frac{H_2 h_2}{B_2} \right)$$

In addition to baseline errors discussed above, an incorrect azimuth will add even more uncertainty. An azimuth error of 0.5° over a baseline of 2.0 km, for example, will produce an error in position of one end of the baseline with respect to the other of 18 meters.

Based on the above analysis, it was initially assumed that the relative horizontal coordinates of T1, T2, and T3 would be known to approximately ± 30 m. Hart (1967), for example, claims that with careful repetitive procedures involving all baselines in a transponder array, relative accuracies of the order of five meters can be achieved.

As preliminary data analysis proceeded with the C10 network, it became obvious that there were major inconsistencies with the given baseline data. Preliminary adjustments (see Chapter V) would not converge when the given transponder coordinates were used. Further analysis suggested both length and azimuth problems with the given T2-T3 baseline. The fact that the T3-T1 baseline had never been determined led to a situation in which no positional redundancy existed in the T1, T2, and T3 network. As a result of these problems, initial positions of T1, T3, and T4 were determined from the Loran coordinates of their drop sites, while T2 was computed relative to T1 from baseline survey data.

As a consequence of this less than ideal procedure, it was considered that standard deviations of ± 100 m should be allocated to the transponder coordinates of T2, T3, and T4, while T1 would be assumed fixed in order to provide a positional datum (albeit a rather arbitrary one) for the array. The 100 m allocated for positional standard deviations seemed reasonable in the light of both the known

positional accuracy of Loran and the added uncertainty from drift as the transponders settled to the bottom.

In summary, therefore, the following standard deviations were assumed:

$$\sigma(T2_{x,y}, T3_{x,y}, T4_{x,y}) = 100 \text{ m}$$

$$\sigma_{T1_{x,y}} = 0.01 \text{ m}$$

B. TIMING

The time observations required a consideration of how well the acoustic time intervals could be established between Pegasus and the transponders. The travel times were assumed to be independent of one another. The Pegasus manual states that the transponders have a 12.5 ms output pulse delay accurate to within ± 0.5 ms (Oceanographic Instrument Systems). For the initial transponder adjustment, $\sigma_{\text{Time}} = 0.002 \text{ s}$ was chosen as a reasonable weight (see Chapter V), and then later reduced to 0.0005 s for the full run with improved transponder coordinates.

$$\sigma_{\text{Time1}} = 0.002 \text{ s}$$

$$\sigma_{\text{Time2}} = 0.0005 \text{ s}$$

C. VELOCITY OF SOUND

Fofonoff (1963) states that velocity of sound is a function of the thermodynamical and chemical state of the water and is determined by using any complete set of variables of state such as temperature, pressure, and salinity. He goes on to observe that the precision of sound velocity must consider not only that of the oceanographic data, but also of the empirical formula used to convert these measurements to sound speed.

Pegasus provides temperature and conductivity data which is converted to a sound velocity profile for the full range of the drop by algorithms published in the Unesco Technical Papers in Marine Science #44 (Fofonoff and Millard, 1983, pp. 11-12, 49).

In dealing with slant ranges, as in this case, acoustic refraction should be considered as well. Spain et al. (1981, pp. 1562-1564) presents an equation derived by Vaas (1964) which corrects for the effects of ray bending, taking into consideration the relative positions of the projector and the receiver even when they are at similar depths. This equation is stated to have an accuracy of better than 0.2 m/s for all angles and depths.

Another excellent source for acoustic refraction information in a survey situation is the SASS Accuracy Study Simplified Ray Bending Correction (General Instrument Corp., 1975) and its follow-up Ray Bending Correction for Depth

Sounders, An Informal Approach (General Instrument Corp., 1976). These two papers develop the equations for an effective sound velocity in detail and include graphs showing the errors relative to lateral angles.

In this study it was found that the use of an average harmonic mean sound velocity produced acceptable results. To obtain this average, the sound velocity profile was used to calculate harmonic sound velocity profiles with respect to each transponder. These four harmonic mean profiles were then averaged (see Figure 12) and the resulting profile used to calculate the approximate X and Y coordinate of each Pegasus position. The harmonic mean profiles differ for each transponder because of the wide difference in deployment depths. Harmonic means at the depths found here differed by about 1.0 m/s from the average used, representing a bias of less than 0.7 m in a typical one km path length.

Refraction was not regarded as significant for this study because of the relatively short path lengths and small sound velocity gradients encountered (Schnebele, 1990b). In deeper waters with longer paths, refraction may be significant.

The harmonic mean used in this model, therefore, assumes no refraction. The small error that it introduces was felt to be inconsequential compared to the uncertainties in transponder coordinates and signal travel time measurements. Thus the harmonic means used in the adjustment were assumed to be exact quantities without significant random error.

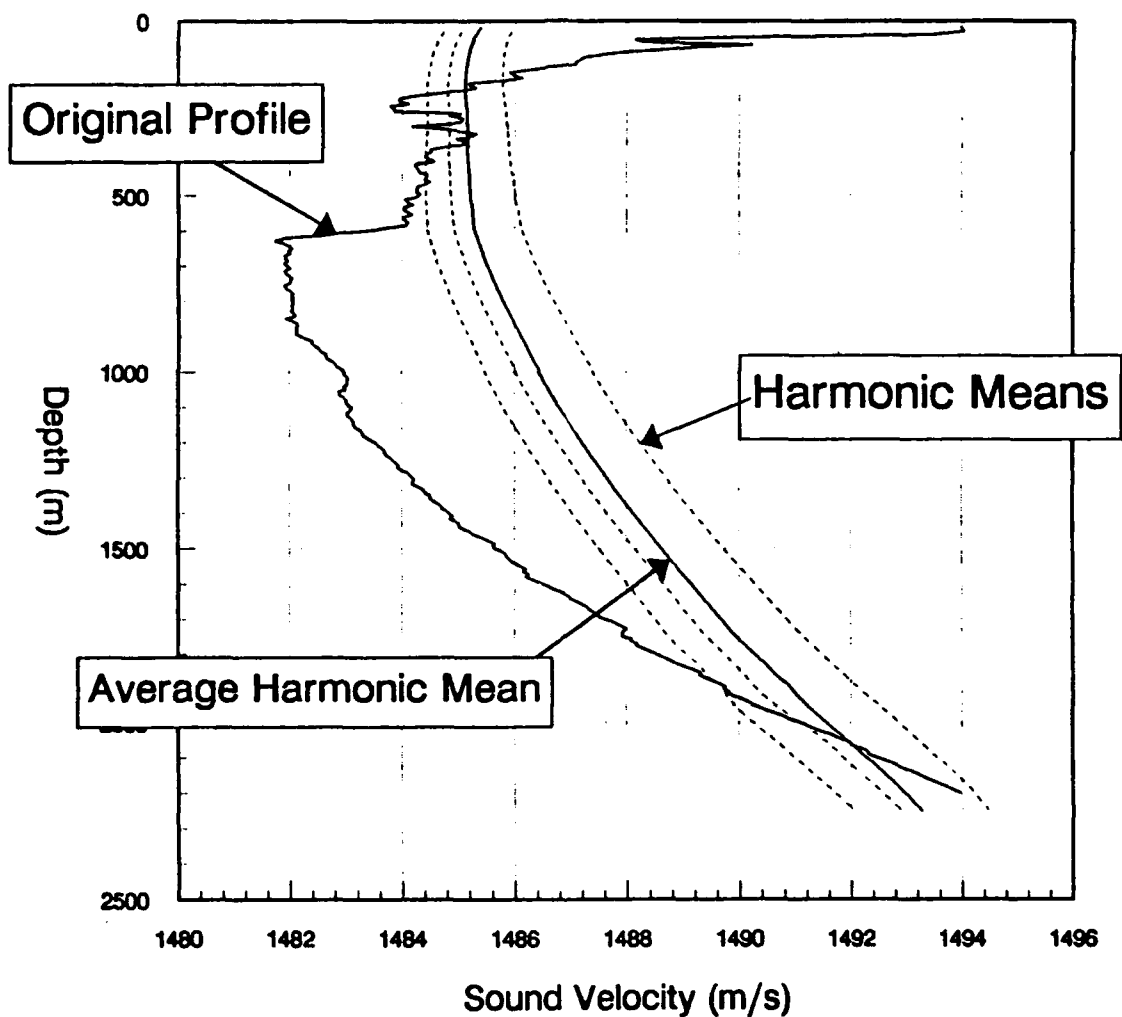


Figure 12. Harmonic Sound Velocity

D. PRESSURE/DEPTH RELATIONSHIP

The algorithm used to convert pressure to depth is the Saunders and Fofonoff formula which uses the hydrostatic equation and the Knudsen-Ekman equation of state (Fofonoff and Millard, 1983, p. 25). The formula includes variation of gravity with latitude and depth and assumes standard sea

water. This formula is said to deviate by only 0.08 meters at 5000 decibars from estimates based on EOS80, a considerably smaller error than those in pressure measurements as shown below.

The manufacturer of the Pegasus pressure transducer claims a typical accuracy under difficult environmental conditions of 0.02% (Paroscientific, 1986). At our full scale of 2200 m, this would compute to about 0.4 m. As the analysis progressed, further inconsistencies in the data began to lead to a suspected bias in pressure measurements, perhaps caused by a temperature hysteresis as Pegasus drops and rises through the deepest part of the water column. Therefore, a $\sigma_{P_z} = 2.00$ m was taken to be the standard deviation of the Pegasus depth measurements, i.e.,

$$\sigma_{P_z} = 2.00.$$

IV. LEAST SQUARES ADJUSTMENT

The specific task of this study was essentially to analyze three basic problems:

- * The precision of the transponder coordinates, especially those of T4, from which the position of Pegasus is derived.
- * The precision of the Pegasus positions and consequently the current velocities derived from Pegasus navigation.
- * The depth at which pressure becomes critical in solving for velocities.

To provide a means of answering these questions, a Least Squares Adjustment Program was developed by Dr. John Hannah.³ This adjustment program, as well as a brief explanation, are provided in Appendix A.

Least squares adjustment techniques are used to determine unique solutions for unknown parameters when there are redundant observations, i.e., more than necessary to specify the model (Davis et al., 1981, pp. 38-39). The least squares estimator is an unbiased minimum variance estimator which is unique, mathematically easy to derive, and, when compared to other estimation techniques, leads to a smaller dispersion of random errors. It also is distribution free in the sense

³Adjunct Research Professor, CNOC Chair in Mapping, Charting, and Geodesy, U.S. Naval Postgraduate School, Monterey, CA., 1988-1990.

that a distribution is needed only for confidence interval testing (Hannah, 1989).

The mathematical model for the Pegasus data adjustment has the general form:

$$L_{a1} = F_1(x_a)$$

$$L_{a2} = F_2(x_a)$$

in which the first set of observations comes from Pegasus itself in the form of return signal travel times from the transponders to the instrument. The second set comes from an *a priori* knowledge of any of the system parameters such as the positions of the transponder coordinates, and each Pegasus depth as calculated from pressure. In general terms, any set of adjusted observations expressed as a function of a set of adjusted parameters can be written in a matrix expression:

$$L_a = F(x_a)$$

The following is adapted from a least squares adjustment procedure written by Hannah (1981) and is used here with the author's permission.

...These functions may be linearized by taking a first order Taylor series expansion about some approximate values for

the parameters, X . The first function then becomes

$$L_b^1 + V_1 = \frac{\partial F_1}{\partial X_a} \bigg|_{X_a=X_0} (X_a - X_0) + F(X_0)$$

or

$$L_b^1 + V_1 = A_1 \hat{X} + L_0^1$$

or

$$V_1 = A_1 \hat{X} + L_1$$

in which

$$A_1 = \frac{\partial F_1}{\partial X_a} \bigg|_{X_a=X_0}, \quad \hat{X} = (X_a - X_0)$$

and

$$L_1 = L_0^1 - L_b^1$$

Similarly, the second function may be linearized to give

$$V_2 = A_2 \hat{X} + L_2$$

In the above, V_1 and V_2 are the vectors of residuals arising from the observations L_b^1 and L_b^2 with the adjusted values of the parameters being given by the vector \hat{X} . Assuming that the two sets of observations are uncorrelated, then the least squares minimum variance estimate of X based on these two sets of observations is given by minimizing the function

$$\phi = V_1^T P_1 V_1 + V_2^T P_2 V_2 - 2K_1^T (V_1 - A_1 \hat{X} - L_1) - 2K_2^T (V_2 - A_2 \hat{X} - L_2)$$

with respect to the unknowns V_1 , V_2 , K_1 , K_2 , and \hat{X} . The weight matrices for each set of observations are given by P_1

and P_2 , in which $P_1 = \Sigma_{L_1}^{-1}$ and $P_2 = \Sigma_{L_2}^{-1}$, i.e. the inverse of the variance-covariance matrices of the observation sets. Infinitely large variances are applied to non-weighted parameters resulting in zeros in the corresponding diagonal elements of P_2 .

After minimizing ϕ and eliminating the unknowns K_1 , K_2 , V_1 , and V_2 , the least squares estimate for X is given by the solution of the normal equations

$$(A_1^T P_1 A_1 + A_2^T P_2 A_2) \hat{X} + (A_1^T P_1 L_1 + A_2^T P_2 L_2) = 0$$

Since, however, the A_2 matrix arises from direct observations on the parameters, the partial derivatives with respect to the parameters $\left. \frac{\partial F_2}{\partial X_a} \right|_{X=X_0} = I$, the identity matrix and thus the above equation reduces to the form

$$(A_1^T P_1 A_1 + P_2) \hat{X} + (A_1^T P_1 L_1 + P_2 L_2) = 0$$

This has the solution

$$\hat{X} = -(A_1^T P_1 A_1 + P_2)^{-1} (A_1^T P_1 L_1 + P_2 L_2)$$

The variance covariance matrix of the parameter estimates is obtained by normal error propagation methods and is given by

$$\Sigma_{\hat{X}_a} = (A_1^T P_1 A_1 + P_2)^{-1}$$

with the *a posteriori* variance of unit weight by

$$\hat{\sigma}_0^2 = \frac{V_1^T P_1 V_1 + V_2^T P_2 V_2}{n_1 + n_2 - u}$$

in which n_1 equals the number of [signal time interval] observations, n_2 equals the number of *a priori* parameter

observations, and u equals the number of parameters in the adjustment....

If the assumptions going into the adjustment are good, then the *a posteriori* variance should be close to 1.0. This statistic is an indicator of the quality of the adjustment.

The least squares adjustment technique is an excellent tool for determining solutions of unknown variables. However, certain limitations do apply. The process assumes that all systematic errors have been removed or accounted for, and that all remaining errors are randomly distributed. Systematic errors that still unaccountably exist in the observations will bias the adjustment such that it may attempt to distribute the errors to all the observations and shift the parameters.

Other factors which may degrade the adjustment are:

- * A poor physical model such as one that ignores scale error.
- * The incorrect weighting of observations.
- * Small residuals which may be a result of poor network geometry or insufficient observations rather than good observations.

For additional information on adjustment computations, see Uotila (1986) for derivations of appropriate expressions for least squares adjustments which use a variety of different systems or groups of observations with their associated constraints.

Appendix A gives greater detail on the least squares adjustment process for this Pegasus data analysis.

V. COMPUTATIONAL PROCEDURE

The data set used for most of the results below was the downcast data from Pegasus Drop #157 on August 3, 1989. Occasional comparison studies were also made with the #157 upcast data, and that of #158 which occurred in the same vicinity later the same day.

As stated in Chapter II, Pegasus data is recorded every 16 seconds as the instrument descends and ascends during a deployment. For this study, the portion of the depth profile of Drop #157 from about 16 m to 2200 m was used, providing 306 Pegasus records. This carried the instrument from surface waters down through the plane of the transponders.

It needs to be stressed at the outset that the computational procedures adopted in this thesis should not be considered as optimum, but rather were developed as processing proceeded in order to overcome difficulties encountered with the specific data set associated with the C10 array. If the problems ultimately discovered with the data set had been known at the beginning, then some procedural points in the data processing would have been slightly revised. This aside, the purpose of the study was to demonstrate the capabilities of least squares estimation procedures to resolve both unknown transponder positions and Pegasus velocity components. As

will be seen in the following discussions, this was more than adequately achieved.

Because of the poor initial positions of the transponders, it was decided to select a small data set of 74 Pegasus records from the total of 306 collected during drop #157 and use these to help provide improved transponder coordinates. This data set was determined by dividing the full depth of the Pegasus drop into 36 intervals and taking a pair of consecutive records in each interval. Using the *a priori* standard deviations derived in Chapter III, an initial solution for the 74 Pegasus positions and the four transponders was obtained. The very small positional standard deviation associated with T1 essentially served to hold this transponder fixed in the resulting solution.

As described in Chapter IV, $(A_1^T P_1 A_1 + P_2)^{-1}$ is the variance-covariance matrix of the adjusted parameters. The upper tridiagonal portion of this matrix is stored in vector form and can be accessed to give information on the variances and covariances of the adjusted parameters as shown in Figure 13.

For example, the portion of this figure which describes Pegasus Position 1 gives the variances of the x, y, and z coordinates, σ_{Px1}^2 , σ_{Py1}^2 , and σ_{Pz1}^2 , respectively. The other three elements in this upper tridiagonal, $\sigma_{Px1 Py1}$, $\sigma_{Px1 Pz1}$, and $\sigma_{Py1 Pz1}$, provide the covariances between the x, y, and z coordinates of the first Pegasus position. Similarly, the

[illegible]

covariances between the coordinates of different positions are provided in their appropriate locations as shown. The variances and covariances of the transponder coordinates are located in the lower right hand corner, and among themselves comprise an upper tridiagonal 12 x 12 submatrix.

The adjustment solution is also used to compute observational residuals which in turn are used to compute an *a posteriori* variance of unit weight. For the 74 record run this *a posteriori* variance turned out to be 0.1196, a very small number compared to that which should ideally have been close to unity. This *a posteriori* variance when multiplied through the variance covariance matrix enables the matrix to be scaled in order that it provide supposedly unbiased estimates of the parameter variances and covariances. This was done with this data set and the resulting transponder coordinates with their newly estimated standard deviations (of the order of 10-20 m) taken and used as *a priori* information in the final solution in which all 306 records were used simultaneously. It was felt that this procedure would enable the transponder coordinates with their associated accuracy estimates to more closely represent the type of situation usually found in a good transponder network.

In retrospect, however, it appears that it may have been more appropriate to have run the 74 point data set with $\sigma_{\text{Time}} = 0.0005$ s rather than the 0.002 s actually used. When, toward

the very end of this study, a σ_{Time} of 0.0005 s was allocated to the measured time delays in the 74 point data set, the *a posteriori* variance of unit weight became 0.96, and the resulting standard deviations on the transponder coordinates approximately 40 m. The coordinate solutions for the transponders did not change by more than 5 m in position although the computed depth of T4 did increase by a further 10 m.

Although this second solution is to be preferred over the first, it was felt that the work and time involved in altering all the results and documentation already completed was not justified, given the minimal impact that it would have on the final results. In fact, it was clear that it would not change any of the conclusions resulting from this study.

Ideally, when dealing with data from a number of Pegasus drops on a single transponder network, it would be best to use a small, but representative (in depth) data set from each drop, and merge these together in a single adjustment to provide an optimum set of transponder coordinates for that array which could, where appropriate, be held fixed in subsequent simultaneous processing of complete drops.

VI. RESULTS OF STATION C10 ADJUSTMENT

A. TRANSPONDER COORDINATES

With improved transponder coordinates and their associated standard deviations determined through the 74 record procedure described in the last chapter, the entire 306 record data set was then run. The results from this full analysis are discussed below.

The final transponder positions were moved (in total) from their original Loran estimates as shown in Figure 14 and Table 2 below.

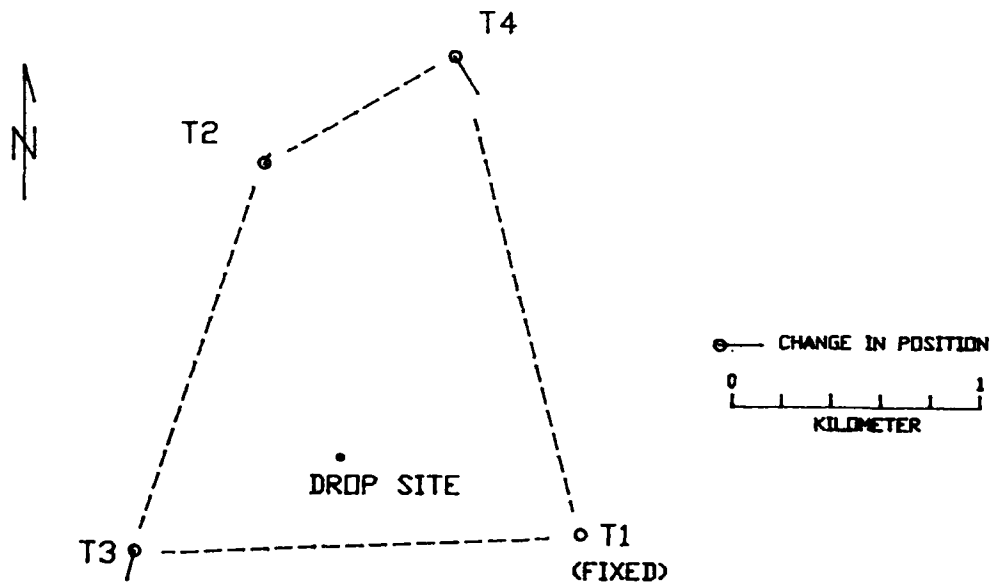


Figure 14. Transponder Movement

TABLE 2
MOVEMENT OF TRANSPONDER POSITIONS

Transponder	X	Y	Z
T1	0	0	4.5
T2	-20.3	-41.9	-6.2
T3	32.2	116.0	-7.1
T4	-90.8	149.3	-6.5

It must be stressed, however, that this data proved to be only internally consistent. Using the same transponder positions (as determined by the 74 Pegasus position data set) to adjust data from another drop #158 (which used only 13 Pegasus positions), physically nearby and later the same day, produced somewhat different transponder coordinates although it converged within itself. The resulting Table 3 is shown below.

It is suspected that the reason for this inconsistency lies both in the very low degrees of freedom in the adjustment of drop #158 (leading to a statistically weak solution) and in the overall weak *a priori* positions of the transponders. In a well-surveyed four transponder array, this problem would not exist.

It appears certain that the location of Pegasus within this weak transponder array has an effect on the final adjusted transponder positions. With the origin held fixed in

TABLE 3
TRANSPONDER COORDINATE SOLUTIONS

	Trans- ponder	X	Y	Z
Drop #157	T1	0	0	1882.86
	T2	-1387.53	1530.88	1987.15
	T3	-1769.38	-66.52	2229.11
	T4	-540.12	2049.71	1893.48
Drop #158	T1	0	0	1879.13
	T2	-1244.88	1502.60	1991.22
	T3	-1779.68	-181.94	2232.52
	T4	-533.65	1950.66	1896.60
Difference Between #157-#158	T1	0	0	-3.73
	T2	-142.65	28.28	-4.07
	T3	10.3	115.42	-3.41
	T4	-6.47	99.05	-3.12

both cases, the array tended to skew in the direction of the drop (see Figure 15). A set of coordinates which would be appropriate for all drops might be obtained by making a series of Pegasus drops out along the edges of the array near the centers of the baselines, in addition to the drop in the center.

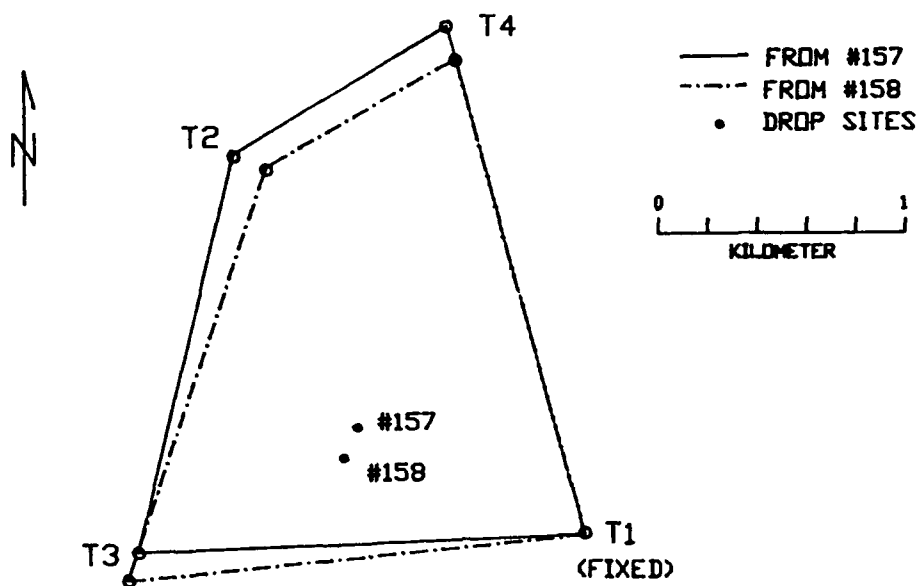


Figure 15. Adjusted Array from #157 and #158 Drops

It is well to note, however, how the transponder positions have been improved over those of our originally assumed coordinates, especially those of T4 whose position was largely unknown (see Table 4).

The horizontal drms values were (when using $\sigma_T = 0.0005$ s):

- * T1 = 0 (held fixed).
- * T2 = 11.2 m.
- * T3 = 13.2 m.
- * T4 = 12.4 m.

These drms values suggest an improved horizontal precision of 13.5 meters or less.

TABLE 4
STANDARD DEVIATION OF TRANSPONDER POSITIONS

	Transponder	X	Y	Z
Initial Standard Deviations	T1	0	0	10
	T2	100	100	10
	T3	100	100	10
	T4	100	100	100
Standard Deviations After Adjustment	T1	0.01	0.01	1.88
	T2	8.32	7.48	1.75
	T3	6.79	11.27	2.03
	T4	11.67	4.12	2.78

B. PEGASUS POSITIONS

The precision of the Pegasus positions is determined from the variance-covariance matrix as described in Chapters IV and V. Typical values at various depths are shown in Table 5.

If geometric studies are desired, these variance-covariance values produce a tri-axial error ellipse for each Pegasus position. For ease of perception, the axes of these ellipses can be converted to an orthogonal system through the use of eigenvalues and eigenvectors. (For a discussion of this process, see Mikhail, 1976.)

TABLE 5
PRECISION OF PEGASUS POSITIONS

Depth (m)	σ_x	σ_y	σ_z
16	4.92	6.32	0.51
248	4.61	6.18	0.49
503	4.31	6.00	0.50
750	3.96	5.80	0.53
1000	3.70	5.62	0.58
1250	3.44	5.56	0.67
1502	3.31	5.52	0.86
1750	3.24	5.16	1.29
2005	3.53	5.06	1.98
2183	3.80	4.76	1.53

C. CURRENT VELOCITIES

Pegasus velocities were determined by using the adjusted values of the Pegasus positions which were determined by the methods described in Chapter V.

It is well to note here the similarities and differences between the Pegasus positions as determined by this adjustment, and those determined by the method currently in use by the NPS Oceanography Department.

The initial Pegasus positions used to start this adjustment had been derived by combining three observation equations into two, and then solving the two equations for the unknown x and y of the Pegasus position. (See procedure explained in Appendix A.) These positions were then refined by the

adjustment process using four transponders where Pegasus depth was constrained by pressure.

The present method used by NPS as described in Chapter II, uses only two observation equations to solve for the positions (i.e., using only two transponders), without further adjustment and without the pressure/depth constraint. Velocities derived from these two sets of positions are compared in Figures 16 and 17. (Both sets of velocities have been filtered by a seven point running average.) Figure 16 plots the U components of adjustment derived velocities against depths, those derived by using the present NPS method, and finally the differences between the two approaches. Similarly, Figure 17 plots comparable information for the V components.

The profiles of the two methods exhibit very similar configurations through much of the range, with notable exceptions occurring at the surface, at mid-range (see discussion of residuals in the section on depth/pressure analysis which follows), and at the bottom of the cast.

The differences in surface velocities between the two methods may be a result of using different sound velocity profiles. The large divergences at depth where the adjusted speeds peak at about 22 cm/s and those of NPS at 111 cm/s, are almost certainly due to a loss of precision in the two-transponder solution, as well as inconsistencies in the

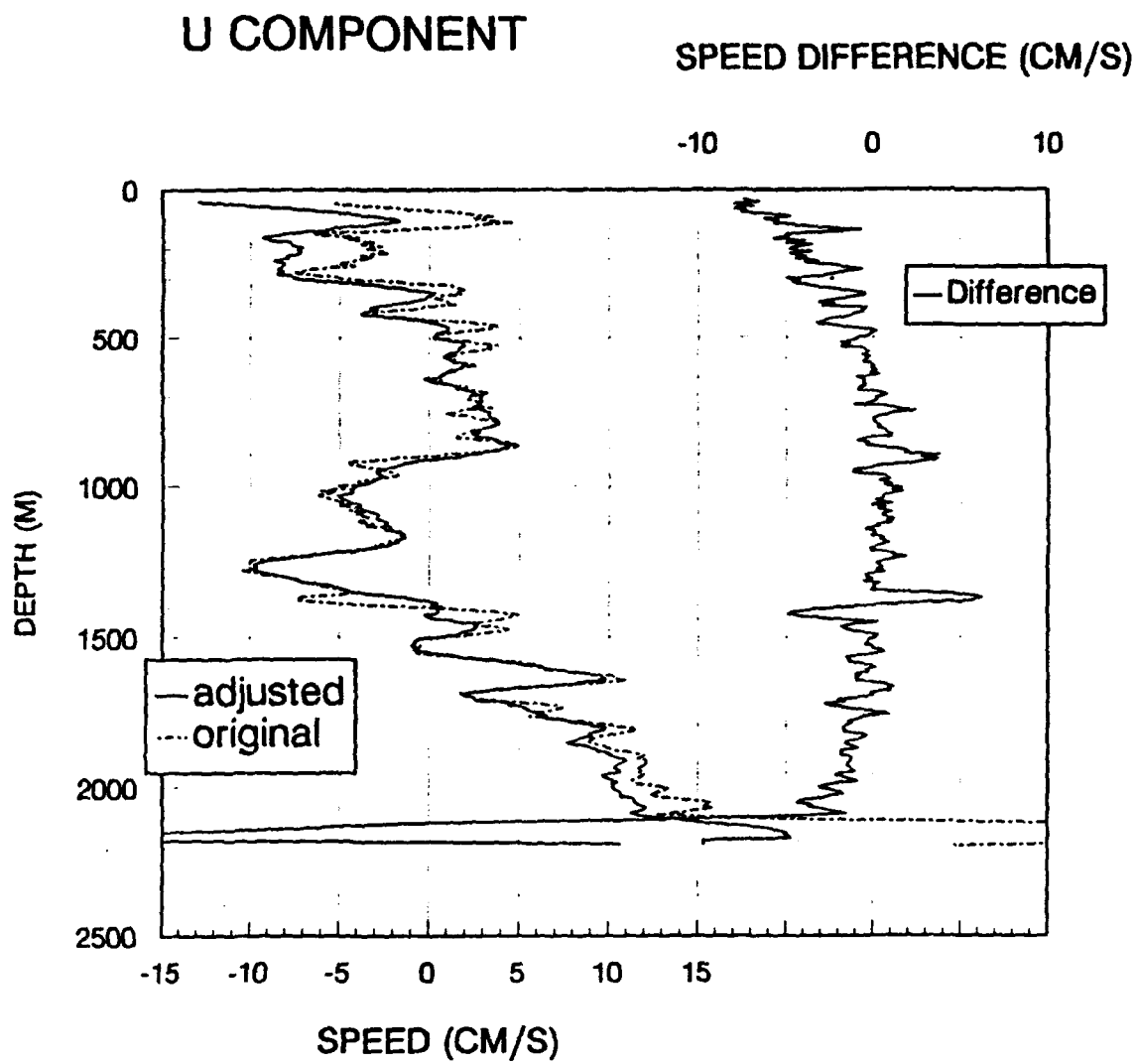


Figure 16. U Component

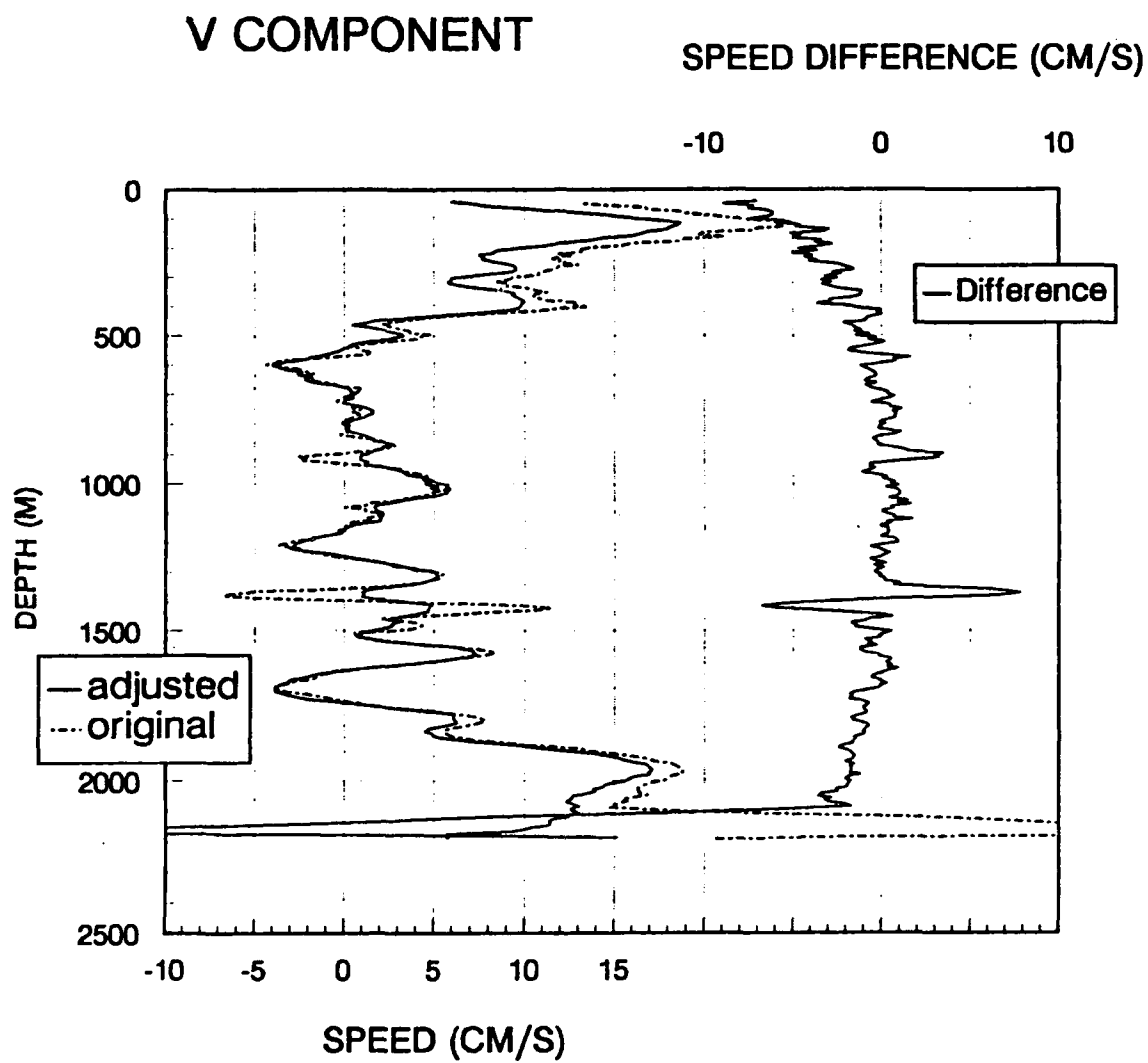


Figure 17. V Component

original pressure measurements. Table 6 provides a sample of the velocities derived by the two approaches, and shows the differences between them. These differences begin to increase markedly at about 2100 m, indicating the substantial role the adjusted transponder network and the inclusion of a pressure observation play in the solution.

TABLE 6
VELOCITY COMPARISONS

Depth (Approx.) (m)	Adjustment derived velocity* (cm/s) (1)	NPS Velocity (cm/s) (2)	Velocity Differ- ence (cm/s) (2)-(1)	Differ- ence in U (cm/s)	Differ- ence in V (cm/s)
40	14.23	13.24	-0.99	+6.30	+5.49
112	18.89	25.41	+6.52	+7.09	+6.29
660	2.10	1.65	-0.45	-0.35	+0.29
1280	10.01	10.74	+0.74	-1.01	-0.77
1380	1.08	9.45	+8.37	-7.06	-7.74
1820	10.34	12.66	+2.33	+1.93	+1.30
2090	17.32	19.49	+2.16	+0.81	+2.20
2112	19.58	35.84	+16.27	+11.61	+11.44
2145	23.07	68.05	+44.98	+33.89	+30.10
2175	16.36	110.75	+94.39	+71.56	+62.95
2190	16.36	49.76	+33.40	+25.57	+22.62

*Adjustment using $\sigma_t = 0.002$, $\sigma_{p_z} = 2$.

Seven point filter used for both sets of velocities.

D. PEGASUS VELOCITY ERRORS

Estimated velocity errors were obtained through the propagation of positional errors found in the variance-covariance matrix of the final run, yielding σ_u , σ_v , and σ_w , the errors in the x, y, and z directions, respectively. This propagation of errors proceeds as follows.

The desired velocity components are given by the expression:

$$\bar{\mathbf{V}} = \begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} (x_2 - x_1) / \Delta t \\ (y_2 - y_1) / \Delta t \\ (z_2 - z_1) / \Delta t \end{bmatrix}$$

where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the positions of Pegasus at respective 16 second intervals (i.e., $\Delta t = 16$ s).

The variance-covariance of $\bar{\mathbf{V}}$ is given by

$$\Sigma_{\bar{\mathbf{V}}} = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} & \sigma_{uw} \\ \sigma_{vu} & \sigma_v^2 & \sigma_{vw} \\ \sigma_{wu} & \sigma_{wv} & \sigma_w^2 \end{bmatrix} = \mathbf{G} \Sigma_{\Delta P} \mathbf{G}^T$$

where

$$\mathbf{G} = \frac{1}{\Delta t} \begin{bmatrix} \frac{\partial}{\partial x_1} & \frac{\partial}{\partial y_1} & \frac{\partial}{\partial z_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial y_2} & \frac{\partial}{\partial z_2} \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

and

$$\Sigma_{\Delta P} = \begin{bmatrix} \Sigma P_1 & \Sigma P_1 P_2 \\ \Sigma P_2 P_1 & \Sigma P_2 \end{bmatrix}$$

In this last expression ΣP_1 and ΣP_2 are the variance-covariance matrices for Pegasus positions 1 and 2, respectively, while $\Sigma P_1 P_2$ and $\Sigma P_2 P_1$ are their cross covariances. These are obtained from the full adjustment variance-covariance matrix described in Chapter V. (Appendix B contains an algorithm for computing the variance-covariance matrix of the current velocities from data obtained from the subroutine MAT in the Least Squares Adjustment program.)

These values provide information on the precision of Pegasus velocity at each position, and change according to the geometry between Pegasus and the transponders. Table 7 provides velocities and their standard deviations at various depths as computed by the adjustment. The horizontal velocity errors are greatest near the surface, and improve as Pegasus descends toward the transponders where, in the horizontal sense, the geometry becomes tighter. For the vertical velocity, the reverse is true.

These velocity errors were disappointing. At the surface, a velocity in the X direction of -17.8 cm/s had a standard error of ± 10.7 cm/s. In the Y direction, a velocity of

TABLE 7
VELOCITIES AND STANDARD DEVIATIONS

Av. Depth (m)	U (m/s)	V (m/s)	W (m/s)	σ_u (m/s)	σ_v (m/s)	σ_w (m/s)
20	-0.1784	0.0169	0.4347	0.1073	0.0822	0.0403
230	-0.0886	0.0789	0.4566	0.0988	0.0759	0.0408
740	0.0236	0.0204	0.4511	0.0795	0.0621	0.045
1233	-0.0729	0.0011	0.4141	0.0643	0.0511	0.0573
1739	0.1178	-0.0238	0.4734	0.0578	0.0439	0.1107
2180	0.1989	0.0681	0.3379	0.0569	0.0454	0.1339

1.7 cm/s had a standard error of ± 8.2 cm/s. At the bottom results were somewhat improved with $u = 19.9$ cm/s and $\sigma_u = 5.7$ cm/s, $v = 6.8$ cm/s and $\sigma_v = 4.5$ cm/s, but not substantially so.

These velocities and error estimates were obtained by looking at only individual pairs of positions. The NPS Oceanography Department computes velocities by using a seven-point running average for each Pegasus recorded depth. This same filtering technique used with the adjusted data would improve precision by $1/\sqrt{7}$ or 0.38, if positions were uncorrelated. However, these positions are correlated, so the expected improvement is somewhat smaller.

There are two ways the precision of the unfiltered velocities could be improved:

- * Use longer time intervals, so that as Δt becomes larger in the error propagation equation discussed above, the variance-covariance matrix $\Sigma \bar{V}$ diminishes.

- * Average the 16-second velocities (in the same manner as NPS) by using a seven-point running average which would improve the precision if the velocities were uncorrelated.

However, both of these techniques, while gaining precision, sacrifice vertical resolution.

There are two important aspects to these formal velocity errors. In the first instance the signal time intervals are only measured to 0.0001 s. At a conventional sound velocity of 1500 m/s this is equivalent to 15 cm in the derived range. This in turn propagates into a velocity error of approximately 1 cm/sec between consecutive Pegasus positions if the transponder positions are assumed to be without error.

In the second instance, the formal errors in the transponder positions propagate directly into errors in the Pegasus positions and thence into the velocity components. These latter errors have by far the most significant influence on the derived standard deviations of the velocity components for Pegasus. This in turn provides an added emphasis on the need for a strong transponder survey.

E. DEPTH/PRESSURE ANALYSIS

To determine at what depth the pressure/depth measurement becomes critical in solving for current velocities, a separate computer run was made with the standard deviation of the Pegasus Z coordinate changed from 2.00 to 50.00 m. The results are graphically illustrated in Figure 18.

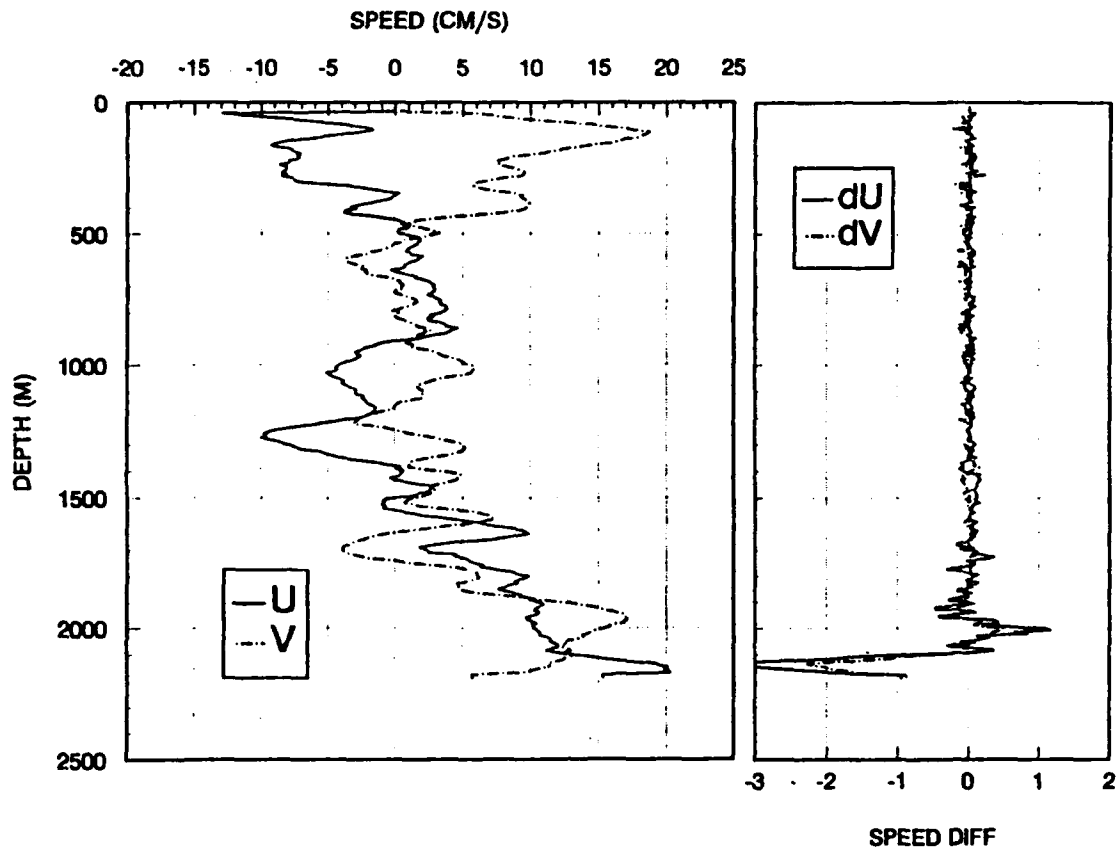


Figure 18. Velocities

Here, the U and V velocity components derived from the adjustment using $\sigma_{p_z} = 2$ m are plotted on the left; speed differences between the U and V components of the two sets of adjustments using $\sigma_{p_z} = 2$, and $\sigma_{p_z} = 50$, respectively, are illustrated on the right. The differences are minimal down through the water column until about 1700 m as Pegasus approaches the plane of the transponder. DU reaches a maximum positive difference of 1.16 cm/s at 2008 m, and subsequent maximum negative difference of -3.53 cm/s at 2135 m.

Respective values for DV are 0.49 cm/s at 2001 m and -2.26 cm/s at 2135 m.

An analysis comparing the standard deviations of Pegasus depth positions shows a large maximum of 36 m at a depth of 2000 m as derived from the free-floating P_z adjustment, compared with a maximum of 2 m at a depth of around 1940 m for tightly-held P_z . The latter reflects the fact that the *a priori* precision estimate for pressure observations was itself 2 m, and that travel time measurement geometry gives no additional information on the adjusted z value.

Table 8 shows observational residuals after both of the adjustments and compares the two sets of residuals at similar depths. (These figures compare runs made with $\sigma_{\text{Time}} = 0.002$ s, rather than the final 0.0005 s.)

At the surface both sets of residuals are small. Where the pressure/depth relationship is tightly constrained, residuals were highest at the bottom of the cast where Pegasus moved through the plane of the transponders, i.e., between about 1870 m to 2240 m. Here the resolution of depth becomes less certain due to the poorer geometry of the Pegasus-Transponder array. These large residuals at depth indicate there may be a bias in the pressure measurements, thus suggesting the presence of a systematic error in the pressure sensor (perhaps due to a hysteresis in temperature readings) that is unaccounted for in the adjustment model.

TABLE 8
OBSERVATIONAL RESIDUALS

Z	σ_{P_z}	T1	T2	T3	T4
Surface	2	-0.0001	-0.0002	0.0001	0.0001
		0.0000	0.0000	0.0000	0.0000
		0.0000	0.0001	-0.0001	-0.0001
	50	0.0000	-0.0002	0.0001	0.0001
		0.0000	0.0000	0.0000	0.0000
		0.0000	0.0001	0.0000	0.0000
1400 m (approx.)	2	-0.0004	-0.0012	0.0006	0.0011
		-0.0005	-0.0013	0.0006	0.0012
		-0.0005	-0.0012	0.0006	0.0011
	50	-0.0004	-0.0011	0.0006	0.0010
		-0.0004	-0.0012	0.0006	0.0012
		-0.0004	-0.0013	0.0007	0.0012
Bottom	2	0.0014	0.0011	0.0007	0.0003
		0.0016	0.0014	0.0008	0.0004
		0.0019	0.0016	0.0008	0.0003
	50	0.0001	0.0001	0.0000	0.0000
		0.0002	0.0002	0.0000	-0.0001
		0.0002	0.0003	-0.0001	-0.0002

Residuals at the bottom of the cast from the adjustment where depth was allowed a very loose constraint are uniformly low. The differences between these two sets of residuals at the bottom, as well as differences in velocities and an increase in σ_{P_z} at this depth as discussed above, all tend to corroborate the suggestion of a bias in the pressure measurements.

Residuals also increased in both cases for a few positions around the depth of 1400 meters. This appeared to occur where the trajectory of the instrument reached its most westerly position. Reasons for this are unclear, although a possible explanation may be related to the fact that this depth and position approximate the location of the shelf edge of the canyon.

It is also possible that one or more of the travel time observations in this part of the drop had large errors. This interpretation is further suggested by the anomalous current shear computed at this depth from the original data (see Figures 16 and 17).

It should additionally be noted that these large residuals at depth and at mid-range seem to display a systematic structure wherein their sign is consistently the same. If the errors were truly random, the positive and negative signs of the residuals would be distributed more or less evenly. This situation is another indication of some systematic error which has not been accounted for in the adjustment model.

VII. CONCLUSIONS AND RECOMMENDATIONS

It is unfortunate that the data set and the transponder array used in this study lacked the quality desirable for detailed analysis. Many of these difficulties were, unfortunately, only discovered as data processing proceeded. It is encouraging to note, however, that the least squares procedures described here enabled the identification of problems with the data which would otherwise have passed undetected.

From a theoretical standpoint, the least squares process must provide better results than the standard NPS positioning techniques because of the fact that it uses all the observational data. It has the added advantage of providing vital statistical information on the quality of the derived results. Unfortunately, in the case of the C10 network, the uncertainty regarding the original positions of the transponders made it difficult to perform a comprehensive comparison between the least squares technique and the standard NPS methodology.

It is observed, however, that the least squares method obtained a solution for transponder positions that converged to less than one meter, with standard deviations that were much smaller than those with which the adjustment started. Precision of the transponder coordinates showed horizontal drms values of less than 15 meters, with standard deviations of transponder depth less than three meters. While these

figures may be optimistic (see Chapter IV), they are indicative of the improvements which can be achieved using these methods.

Regarding the precision of the horizontal Pegasus positions, the standard deviation of the X values ranged from 4.9 down to 3.2 m, reaching a minimum around a depth of 1800 m just at the upper limit of the transponder plane, and then increasing slightly to the bottom of the cast. Pegasus Y values showed standard deviations of 6.3 to 4.8 m, with passage through the transponder plane not as noticeable.

Standard deviations of the Pegasus Z values, when they were tied tightly to pressure, ranged from 1.4 m near the surface to a maximum of 2.0 m at a depth of 1943 m. When the depth was only loosely constrained, standard deviations varied from 2.3 m near the surface, to 8.6 around depths of 1800 m just prior to entering the plane of the transponders, and reached a maximum of 36.5 at a depth of 2000 m. From there they steadily diminished back to 10.4 at the final depth of 2165 m.

Comparison of the results of the NPS method and the least squares method shows a considerable difference in velocities at depths where Pegasus passes through the plane of the transponders. This difference at depth occurs also in velocity comparisons between adjustments made holding depth constrained with a standard deviation of 2.0 m, and allowing it to float more freely with a standard deviation of 50.0 m.

This leads to a suspicion that there may be a systematic error in the pressure reading that has not been accounted for in the model, perhaps due to a hysteresis in temperature readings. Accuracy of pressure observations becomes critical at depths below 1700 m.

It was disappointing to note that the standard deviations on the computed Pegasus velocities (when using $\Delta t = 16$ sec) were often of a similar magnitude as the velocities themselves (i.e., 5-10 cm/s). However, with resolution of signal travel time possible only to the nearest 0.0001 s, resolution of range becomes ± 15 cm. This in turn propagates into velocity errors in the order of 1-2 cm/sec. Therefore, it is unrealistic to look for much greater precision than that.

It is recommended that:

- * NPS refine the existing least squares techniques and the adjustment software to facilitate its use on a regular basis, for production operations.
- * Four transponders be used in each network. This will both strengthen the solutions for the Pegasus velocities and provide a reasonable measure of redundancy for the adjustment procedure.
- * More acceptable positioning of transponders be undertaken, within the time constraints involved, including observation of all baselines. Close attention should be paid to both length and azimuth.
- * That a small, but well-distributed data set of Pegasus records be used from each drop to assist in providing a unique set of transponder positions for each network and that these positions be held fixed in the subsequent adjustment of the full set of records from each drop.
- * Where practical, pressure information always be collected. It both adds to the overall redundancy in the

network and enables additional parameters to be introduced if necessary.

* A very well-controlled set of experimental data be collected on a well-positioned five transponder network. This will enable clarification of the following issues:

- The lack of consistency between the adjusted transponder positions when using different data sets for the same area.
- The question of pressure/depth relationships, especially at depth.
- Possible causes for high residuals at a given depth (1400 meters in the case of station C10).
- The introduction of additional parameters to describe possible mis-calibration of the pressure head or hysteresis of temperature readings.
- The investigation of the actual motion of Pegasus as it descends and ascends.
- The geometric impact of drops made on the edges of the transponder array as opposed to those made at the center of the array.

APPENDIX A

LEAST SQUARES ADJUSTMENT PROGRAM

The least squares adjustment equation for a combined system of two sets of observations is (Uotila, 1986, p. 97):

$$\hat{x} = -(A_1^T P_1 A_1 + A_2^T P_2 A_2)^{-1} (A_1^T P_1 L_1 + A_2^T P_2 L_2)$$

where, in the terminology of the program:

\hat{x} = column vector of corrections to the parameters
(NP,1);

L = Observations (NOBS,1);

A = Jacobian matrix, or the partial derivatives of each observation with respect to each parameter
(NOBS,NP);

A^T = Transpose of A (NP,NOBS);

P = Weight matrix (NOBS,NOBS);

N = Maximum # of Pegasus positions to be determined;

NT = Maximum # of transponders in the network;

NOBS = Maximum # of observations

NP = Maximum # of parameters allowed = (N × 3) +
(NT × 3).

A. FIRST SET OF OBSERVATIONS

1. L₁ Matrix

The first set of observations uses one-way travel times from the transponders to Pegasus as the observations. Parameters are the x, y, and z coordinates of each Pegasus position.

$$L_1 = F_1(x)$$

$$\text{Time} = \text{Distance}/\text{Velocity}$$

Position 1:

$$\text{Time}_1 = [(XP_1 - X_1)^2 + (YP_1 - Y_1)^2 + (ZP_1 - Z_1)^2]^{\frac{1}{2}} / V$$

$$\text{Time}_2 = [(XP_1 - X_2)^2 + (YP_1 - Y_2)^2 + (ZP_1 - Z_2)^2]^{\frac{1}{2}} / V$$

$$\text{Time}_3 = [(XP_1 - X_3)^2 + (YP_1 - Y_3)^2 + (ZP_1 - Z_3)^2]^{\frac{1}{2}} / V$$

$$\text{Time}_4 = [(XP_1 - X_4)^2 + (YP_1 - Y_4)^2 + (ZP_1 - Z_4)^2]^{\frac{1}{2}} / V$$

·
·
·

Position N:

Time_i = time intervals from transponders 1,4;

V = sound velocity;

XP, YP, ZP = Pegasus position coordinates;

X_i, Y_i, Z_i = transponder coordinates;

Observations = $N \times 4$.

Each Pegasus position therefore has four time-interval observations, one for each transponder. Thirteen Pegasus positions will provide 52 time observations and (13×3) parameters, and therefore 13 degrees of freedom for this first set of observation equations. For each additional Pegasus position added to the data set, an additional degree of freedom is gained. If *a priori* constraints are introduced for the transponder coordinates and the transponder positions solved for in the adjustment (as has been done here), then the number of observations and the number of unknowns rises by 12.

The adjustment procedure requires an *a priori* knowledge of the transponder positions, the depth of each Pegasus position (derived from the pressure information) and the sound velocity for each Pegasus position. Estimates of the x and y coordinates are obtained by taking the three observation equations for each Pegasus position that contain information from transponders T1, T2, T3 (presumably the best known). Subtracting the first equation from the second, and the second from the third, will yield two equations in two unknowns, and therefore a resulting first guess solution for XP and YP at that position.

The L_1 matrix in the adjustment equation is a matrix of differences between observed time intervals and what the observation equations would yield using our first guess parameters.

$$L_1 = L_1 \text{ calculated} - L_1 \text{ observed}$$

2. A_1 Matrix

The A_1 matrix contains the derivatives of each observation with regard to each parameter:

$$A_1 \text{ Matrix} = \frac{\partial F}{\partial x_a}$$

If

$$T_1 = [(XP_j - X_i)^2 + (YP_j - Y_i)^2 + (ZP_j - Z_i)^2]^{\frac{1}{2}} / V = D/V$$

$$i = 1, 2 \dots 4$$

$$j = 1, 2 \dots N$$

Then

$$\frac{\partial T_1}{\partial XP_1} = \frac{(XP_1 - X_1)}{DV}$$

·
·
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$$\frac{\partial T_4}{\partial ZP_N} = \frac{(ZP_N - Z_4)}{DV}$$

The derivatives of the transponder positions will be the negatives of the corresponding Pegasus position derivatives.

$$\frac{\partial F_i}{\partial X_i} = - \frac{(XP_j - X_i)}{DV}$$

This will create a matrix in the form:

		$\partial/\partial p_1$	$\partial/\partial p_2$		$\partial/\partial T_1$	$\partial/\partial T_2$	$\partial/\partial T_3$	$\partial/\partial T_4$
Peg. Pos 1	t ₁	x,x,x		x,x,x
	t ₂	x,x,x		x,x,x
	t ₃	x,x,x		x,x,x
	t ₄	x,x,x		x,x,x
Peg. Pos 2	t ₁		x,x,x	x,x,x
	t ₂		x,x,x	x,x,x
	t ₃		x,x,x	x,x,x
	t ₄		x,x,x	x,x,x
Peg. Pos N	t ₁			
	t ₂			
	t ₃			
	t ₄			

where:

$$t_i = \text{Time}_i;$$

$$p_j = XP_j, YP_j, ZP_j;$$

$$T_i = X_i, Y_i, Z_i;$$

$$i = 1, 2 \dots 4;$$

$$j = 1, 2 \dots N.$$

If submatrices are described by

$$a_j = \begin{bmatrix} x, x, x \\ x, x, x \\ x, x, x \\ x, x, x \end{bmatrix} \quad S_j = \begin{bmatrix} x, x, x, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, x, x, x, 0, 0, 0, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, x, x, x, 0, 0, 0 \\ 0, 0, 0, 0, 0, 0, 0, 0, 0, x, x, x \end{bmatrix}$$

Then the A_1 matrix becomes

$$\begin{bmatrix} a_1 & 0 & 0 & \dots & 0 & S_1 \\ 0 & a_2 & 0 & \dots & 0 & S_2 \\ 0 & 0 & a_3 & \dots & 0 & S_3 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & a_N & S_N \end{bmatrix}$$

For each Pegasus position there will be a 4×3 block for the Pegasus derivatives and a 4×12 block for the transponder derivatives. All other positions in the matrix will be zero. Therefore, to drastically reduce computer

memory and time requirements, the program was developed to run in a sparse matrix form whereby only the non-zero elements are used. To do this, the whole A matrix is divided into parts:

- * A--which contains the values for the Pegasus components.
- * S--which contains those for the transponders.

3. P₁ Matrix

P₁, the weight matrix, is set up as a diagonal matrix and stored in column vector form, assuming no covariances between observations. If it is desirable to add covariances, then an error propagation subroutine can be inserted to fill out the weight matrix and the program altered accordingly. In our case the chosen standard deviation for the time interval was 0.002 s, later changed to 0.0005 s. It was entered into the program in the error propagation section where the code reads:

```
Do 35 I = 1,NT
P(1,1) = 1.0D0/0.0005D0**2
35 Continue
```

B. SECOND SET OF OBSERVATIONS

1. L₂ Matrix

The second set of observations corresponds to various parameters which have been observed, in this case the position coordinates of the four transponders, and the position of the Z coordinate of each Pegasus position which allows the depth/pressure relationship to be constrained.

$$L_2 = F(x)_2$$

$$X_1 = X_1$$

$$Y_1 = Y_1$$

.

.

X,Y,Z of each transponder

.

$$Z_4 = Z_4$$

$$ZP_i = ZP_i$$

Depth of Pegasus as computed

$$i = 1, N$$

from pressure.

As described above, we already have first guess estimates of all these observations. The L_2 matrix will initially be zero since for the first solution, the observations equal the parameters, i.e., $X_1 = X_1$. However, with further iterations, this will not be the case.

2. A_2 Matrix

The A_2 matrix contains the derivatives of each observation with regard to each parameter:

$$A_2 = \frac{\partial F}{\partial x_a}$$

For example:

$$\frac{\partial x_1}{\partial x_1} = 1$$

$$\frac{\partial x_1}{\partial \text{all else}} = 0$$

This produces the following diagonal matrix of 1's and 0's:

[illegible]

3. P₂ Matrix

P_2 is a similar diagonal matrix stored in column vector form. It weights all Pegasus z positions to allow for tightening or loosening the depth/pressure relationship. This chosen standard deviation, in our case a value of 2.00, enters the program through an interactive request at the beginning of the program run. The standard deviation of each transponder

position coordinate is read into the program from the original data set. The P_2 matrix will appear as:

$$\begin{bmatrix} 0 & & & & & & & & \\ & 0 & & & & & & & \\ & & 1/\sigma_{P_{z1}}^2 & & & & & & \\ & & & 0 & & & & & \\ & & & & 1/\sigma_{P_{z2}}^2 & & & & \\ & & & & & \ddots & & & \\ & & & & & & 1/\sigma_{x1}^2 & & \\ & & & & & & & 1/\sigma_{y1}^2 & \\ & & & & & & & & \ddots & \\ & & & & & & & & & 1/\sigma_{z4}^2 \end{bmatrix}$$

C. MATRIX OPERATIONS

Now all the matrices are in place and matrix operations can begin.

$$\hat{x} = -(A_1^T P_1 A_1 + A_2^T P_2 A_2)^{-1} (A_1^T P_1 L_1 + A_2^T P_2 L_2)$$

Since the A_2 matrix is a diagonal of 0's and 1's, the second term in the first parenthesis merely adds weight to the appropriate diagonal position in the first term.

D. REQUIREMENTS FOR RUNNING THE PROGRAM

1. Initial Steps

Before the program is run, the following steps must be taken:

- * Variables must be dimensioned adequately, following the definitions clearly stated in the preface to the program.
- * The value of σ_{Time} must be defined (in our case 0.0005), and/or the appropriate error propagation subroutine added.
- * A data set file must be available for access and set up in the following fashion:
 - Transponder coordinates, each with its own standard deviation.
 - For each Pegasus position: pressure, depth, four transponder one-way time intervals, and sound velocity. (Sample data set is shown below.)

Transponder Coordinates	T1	0.0	0.0	1878.4				
	T2	-1324.6	1485.8	1993.4				
	T3	-1401.6	-182.5	2236.2				
	T4	-449.3	1900.4	1900.0				
Transponder Standard Deviations	T1	00.01	00.01	10.00				
	T2	50.00	50.00	10.00				
	T3	50.00	50.00	10.00				
	T4	50.00	50.00	100.00				
Pegasus Positions	1	30.9	30.6	1.4145	1.5486	1.6277	1.6803	1485.310
	2	213.4	211.7	1.3255	1.4245	1.5191	1.5704	1485.100
	3	398.0	394.7	1.2356	1.3083	1.4105	1.4712	1485.140
	4	581.5	576.4	1.1414	1.2047	1.3062	1.3844	1485.250
	.	764.7	757.7	1.0457	1.1140	1.2043	1.3106	1485.650
	.	944.0	934.9	0.9592	1.0282	1.1121	1.2403	1486.200
	.	1125.8	1114.5	0.8941	0.9381	1.0179	1.1754	1486.820
	.	1306.1	1292.4	0.8405	0.8631	0.9243	1.1302	1487.570
	.	1488.7	1472.5	0.7990	0.7944	0.8472	1.0851	1488.440
	.	1667.0	1648.1	0.7620	0.7455	0.7922	1.0491	1489.370
	.	1846.2	1824.5	0.7341	0.7229	0.7534	1.0306	1490.400
	.	2021.9	1997.4	0.7337	0.6973	0.7584	0.9940	1491.510
	.	2200.7	2173.1	0.7415	0.6950	0.7910	0.9671	1492.760
		Press- ure	Depth	T ₁	T ₂	T ₃	T ₄	Sound Velocity
Time Intervals								

* FILEDEFS for input and output files must be in place, either typed in before the program starts or available through a program exec. The program will run in either WF77 or FORTVS2 fortran.

- FILEDEF 01 DISK fn ft fm (i.e., Test Data A).
- FILEDEF 02 DISK(recfm vb lrecl 132 blksize 134).
- (For large data sets, use FILEDEF 02 DISK(recfm fb lrecl 132 blksize 13200).

* Be sure enough memory is available. Two M sufficed for running at least up through 74 Pegasus positions, but 4096 K were required for the 306 position run.

The program starts off by asking for operator input (sample answers in parentheses), requesting:

- * Project or run description.
- * Number of transponders (4).
- * Global standard deviation to be given to the Pegasus z coordinates (2.00).
- * Convergence limit for adjustment and number of iterations (entered separately) (1.000), (4).

From then on the program works on its own, finally producing whatever output has been requested in the output file.

2. Brief Program Outline

- * Matrices are zeroed out at appropriate times.
- * Pegasus data set is read in.
- * Subroutine APPRO is called to calculate initial approximate coordinates of Pegasus.
- * The diagonal weight matrix P_1 is set up and stored in matrix BIGP.
- * Program constants are calculated.
- * Weight matrix P_2 is generated.
- * The L_1 matrix is formed.
- * The matrices A and S are formed by calling subroutine SETUP.
- * The normal matrix equations are formed block by block by calling subroutine NORM. Outputs include the normal matrix (ATPA), in column vector form called anorm, and xhat the ATPL vector called xhat (which subsequently becomes the solution vector).
- * Subroutine P2L2 is called to compute the P_2L_2 matrix and add it to the column vector. It also increments the diagonal elements of the normal matrix to include the influence of P_2 .
- * The normal equations are solved by calling a user supplied routine that will accomplish the required matrix

operations for producing the adjusted values with which to correct the parameters. The IMSL routine LINV3P was used in this study. Utilizing a symmetric storage mode, this algorithm replaces the ATPA matrix by its inverse which is the variance-covariance matrix of the Pegasus and transponder positions. This $(ATPA)^{-1}$ matrix can be written to file for later access in order to produce the error matrix required for current velocity analysis (see Chapter V) as used in the program provided in Appendix B.

- * The parameter corrections are tested to see if iteration is required. If so, the program then updates the parameters and goes through the process again for as many iterations as are called for, or until the convergence limit has been reached.
- * Residuals and the *a posteriori* variance of unit weight are computed by calling subroutine RESID. (Residuals here are, of course, given in units of seconds since they relate to time interval observations.)

3. Program Outputs

Outputs include:

- * Original data set and interactive information given at the beginning of the program.
- * Adjusted parameters following each interaction.
- * Residuals and *a posteriori* variance of unit weight.
- * Any other information requested to be written to a file.

For this study a piece of code, Subroutine MAT, was attached at the end of the program to pick out from the anorm vector only the data required for current velocity analysis. This subroutine is called after the write statement for the *a posteriori* variance of unit weight which uses format statement #175. The anorm vector is the upper tridiagonal of the $(ATPA)^{-1}$ variance-covariance matrix stored in column vector form. (See the diagram in Chapter V.)

The position of the variance of each parameter can be obtained by:

$$\frac{N \times (N + 1)}{2}$$

where N is the place number of the parameter. For example, suppose the variance for the Pegasus y coordinate in the second record was requested. YP₂ is the 5th parameter--XP₁, YP₁, ZP₁, XP₂, YP₂, ..., (5*6)/2 = 15. The variance for YP₂, therefore, will occupy the 15th place in the anorm vector.

	XP ₁	YP ₁	ZP ₁	XP ₂	YP ₂	ZP ₂	
	1	2	4	7	11	16	. . .
Peg. Pos. 1		3	5	8	12	17	. . .
			6	9	13	18	. . .
				10	14	19	
Peg. Pos. 2					(15)	20	. . .
						21	. . .

The subroutine Mat picks up only those values in the anorm vector that fill the 3 x 3 variance-covariance matrix of each Pegasus parameter, and the 3 x 3 covariance portion that relates each parameter to the next one in line. A sample of the output follows:

Var-covar	24.2407314396153941	10.6320080410266140	0.934616249631990276E-01
Pos 1	10.6320080410266140	39.0923498357777104	-0.591505777505010405
	0.934616249631990276E-01	-0.591505777505010405	0.260716105532003254
Covar	22.7007923927003546	10.5979145727731243	0.631260231304605242E-01
Pos 1-Pos 2	10.5612029360100449	39.0364303165567090.	-0.446420407799607621
	0.031007664195040032E-01	-0.439529061104100030	0.520710922203657129E-01
Var-covar	24.0999750884637062	10.6090702710410938	0.723917462167040005E-01
Pos 2	10.6090702710410938	39.9102237605932366	-0.595096665108637957
	0.723917462167040005E-01	-0.595096665108637957	0.260009015573909333
Covar	22.5671757193201941	10.5709158904519001	0.405779410760951404E-01
Pos 2-Pos 3	10.5509507359560424	39.0494970092177121	-0.445954216383701299
	0.636726100041090147E-01	-0.443499422139607530	0.510057407933755941E-01
Var-covar	23.9640764006009367	10.5943499042046341	0.565971236676037024E-01
Pos 3	10.5943499042046341	39.9090236950414461	-0.593990610935371994
	0.565971236676037024E-01	-0.593990610935371994	0.259193690776073099

The first set of three lines with three values each is the variance-covariance matrix for Pegasus position 1, which is symmetric. The second three lines of three values is the covariance matrix for position 1 and position 2 (nonsymmetric). The third set of three lines is the variance-covariance matrix for Pegasus position 2 (symmetric), and so on.

The final 30 lines in the output provide the variances of the transponder coordinates and all the covariances between them.


```

C  @      UNITS 5 AND 6 READ AND WRITE FROM AND TO THE SCREEN      @
C  @      FOR INTERACTIVE PROCESSING. UNITS 1 AND 2 READ AND      @
C  @      WRITE FROM AND TO DISK FILES.                            @
C  @                                                                @
C  @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C
C      WRITE(6,*) 'PROJECT OR RUN DESCRIPTION (TO 60 CHARACTERS)'
C      READ(1,10) PROJ
10  FORMAT(A12)
C      WRITE(6,*) 'NUMBER OF TRANSPONDERS (THEIR COORDINATES AND STD. ',
1   'DEVIATIONS TO BE READ FROM THE DISK FILE)'
C      READ(1,*) NT
C      READ(1,*) (X(I),Y(I),Z(I), I=1,NT)
C      READ(1,*) (SDX(I),SDY(I),SDZ(I), I = 1,NT)
C      WRITE(6,*) 'THE GLOBAL STD. DEVIATION TO BE GIVEN TO THE ',
1   'Z PEGASUS COORDINATES'
C      READ(1,*) SDZP
C      WRITE(6,*) 'CONVERGENCE LIMIT FOR ADJUSTMENT AND # OF ITERATIONS'
C      READ(1,*) TOL, NITER
C      WRITE(2,15) PROJ,NT,(X(I),SDX(I),Y(I),SDY(I),Z(I),SDZ(I),I =1,NT)
15  FORMAT('1',///,5X,70('*'),//,10X,A60,///,5X,70('*'),//,10X,
1   'NUMBER OF TRANSPONDERS =',I2,///,'TRANSPONDER COORDINATES AND ',
2   'THEIR STANDARD DEVIATIONS',//,4(10X,3(F10.2,2X,F6.2),/),///)
C      WRITE(2,16) SDZP
16  FORMAT(' ',5X,'THE GLOBAL STANDARD DEVIATION FOR THE PEGASUS ',
1   'Z COORDINATES =',F7.2)
C      WRITE(2,17) TOL, NITER
17  FORMAT(' ',//,5X,'THE CONVERGENCE LIMIT FOR THE ADJUSTMENT =',
1   'F7.3,//,6X,'THE NUMBER OF ITERATIONS ALLOWED =', I3,///,
2   50X,'INPUT DATA',//,5X,'PRESS.',4X,'ZP      TIME DELAY 1',2X,
3   'TIME DELAY 2',2X,'TIME DELAY 3',2X,'TIME DELAY 4', 5X,'SOUND ',
4   'VELOCITY',2X,'APPROX. PEGASUS POSITIONS(X,Y,Z)',/)
C
C  @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C  @                                                                @
C  @      READ IN THE PEGASUS DATA, POSITION BY POSITION            @
C  @                                                                @
C  @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C
C      FIRST ZERO OUT THE P MATRIX
C
C      DO 20 I = 1,NT
C      DO 20 J = 1,NT
C      P(I,J) = 0.0DO
20  CONTINUE
C      N = 0
25  READ(1,*, END = 45) PRESS, ZP,(TRANS(I), I=1,NT),SV(N+1)
C      NNT = N*NT
C      DO 30 I = 1,NT
C      LB(NNT + I,1) = TRANS(I)
30  CONTINUE
C
C      CALCULATE THE APPROXIMATE POSITION OF PEGASUS
C

```



```

      K = 1
      J = N*3 + 1
      DO 66 I = J, NP, 3
        X(K) = XO(I)
        Y(K) = XO(I+1)
        Z(K) = XO(I+2)
        K = K+1
66 CONTINUE

C
C
C BEGIN THE FORMATION OF THE A AND L1 MATRICES
C
      DO 100 K = 1, NMAX
        JJ = (K-1)*NT
        DO 68 I = 1, NT
          TRANS(I) = LB(JJ + I, 1)
          DO 68 J = 1, NT
            P(I, J) = BIGP(I, JJ + J)
68 CONTINUE
        KK = (K-1)*3
        XP = XO(KK + 1)
        YP = XO(KK + 2)
        ZP = XO(KK + 3)

C
C
      DO 70 I = 1, NT
        L1(JJ + I, 1) = (DSQRT((XP-X(I))**2 + (YP-Y(I))**2 + (ZP-Z(I))**2)
1 /SV(K)) - TRANS(I)
        L(I, 1) = L1(JJ + I, 1)
70 CONTINUE

C
C ZERO OUT THE A AND S SUBMATRICES
C
      DO 80 I = 1, NT
        DO 75 J = 1, 3
          A(I, J) = 0.0D0
75 CONTINUE
        DO 80 J = 1, NT3
          S(I, J) = 0.0D0
80 CONTINUE

C
      CALL SETUP(X, Y, Z, XP, YP, ZP, SV(K), K, NT, NV, NT3, NMAX, A, S, ICOL, IROW,
1 VALUE)

C
      CALL NORM(A, S, P, L, K, NT, NT3, NN, NP, NMAX, ATP, ATPA, ATPS, ATPL, STP,
1 STPS, STPL, ANORM, XHAT)
100 CONTINUE

C
      WRITE(2, 804)(L1(I, 1), I=1, NOBS)
804 FORMAT('1', ///, 50X, 'L1 MATRIX', ///, 8(1X, 10(F10.7, 2X)), 1X, 2F10.7)

C
C PLACE SPS AND SPL INTO THE NORMAL MATRIX AND THE COLUMN VECTOR
C RESPECTIVELY.
C
      DO 110 I = 1, NT3
        NCOL = (NP + 1) - I

```

```

      I1 = NCOL*(NCOL + 1)/2 + I
      DO 110 J = I,NT3
      I2 = I1 - J
      ANORM(I2) = SPS(NT3-J+1,NT3-I+1)
110  CONTINUE
      I3 = NMAX*3 + 1
      DO 120 I = I3,NP
      XHAT(I) = SPL(I-I3+1,1)
120  CONTINUE
C
C   SET UP THE L2 MATRIX AND ADJUST THE DIAGONAL ELEMENTS OF THE
C   NORMAL MATRIX FOR THE INFLUENCE OF P2
C
      CALL P2L2(P2,LB2,XO,ANORM,L2,XHAT,NP,NN)
C
C   @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C   @
C   @   SOLVE THE NORMAL EQUATIONS AND COMPUTE THE RESIDUALS   @
C   @
C   @@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C
C   CALL A USER SUPPLIED ROUTINE TO SOLVE THE NORMAL EQUATIONS.
C   (SEE APPENDIX A, SECTION D.)

      DO 135 I = 1,NP
      XHAT(I) = -XHAT(I)
      XA(I) = XO(I) + XHAT(I)
      XO(I) = XA(I)
135  CONTINUE
      ITER = ITER + 1
      WRITE(2,140)
140  FORMAT('0',///,5X,'ADJUSTED PEGASUS POSITIONS (WITH THE ',
1      'CORRECTIONS TO THE APPROX. POSITIONS IN PARENTHESES)',//, 18X,
2      'X',18X,'Y',18X,'Z')
      J = NP - NT3
      DO 145 I=1,J,3
      WRITE(2,142) XA(I),XHAT(I),XA(I+1),XHAT(I+1),XA(I+2),
1      XHAT(I+2)
142  FORMAT(' ',10X,F9.3,2X,(' ',F6.1,')'),2(2X,F9.3,2X,(' ',F6.1,')'))
145  CONTINUE
      J = J + 1
      WRITE(2,146)
146  FORMAT(' ',//,5X,'ADJUSTED TRANSPONDER POSITIONS (WITH',
1      'CORRECTIONS TO THE APPROX. POSITIONS IN PARENTHESES)')
      DO 148 I=J,NP,3
      WRITE(2,142) XA(I),XHAT(I),XA(I+1),XHAT(I+1),XA(I+2),
1      XHAT(I+2)
148  CONTINUE
C
C   TEST THE PARAMETER CORRECTIONS TO SEE IF ITERATION IS REQUIRED
C
      IFLAG = 0
      DO 150 I =1,NP
      IF(DABS(XHAT(I)).GT.TOL)IFLAG = IFLAG + 1
150  CONTINUE

```



```

C          X,Y,Z - ARRAYS CONTAINING THE COORDINATES OF THE TRANSPONDERS
C          SV    - VELOCITY OF SOUND IN WATER
C          NT    - NUMBER OF TRANSPONDERS
C          OUTPUT: XP,YP,ZP - COORDINATES OF PEGASUS
C
C          DIMENSION TRANS(NT), X(NT),Y(NT),Z(NT),AUX(2)
C
C          DO 10 I = 1,NT
C            TRANS(I) = TRANS(I)*SV
10 CONTINUE
C          DO 20 I = 1,2
C            AUX(I) = (TRANS(I+1)**2 - (ZP-Z(I+1))**2 - X(I+1)**2 - Y(I+1)**2)
C            1 - (TRANS(I)**2 - (ZP-Z(I))**2 - X(I)**2 - Y(I)**2)
20 CONTINUE
C            YP = 0.5D0*(AUX(1)/(X(1)-X(2)) - AUX(2)/(X(2)-X(3)))/((Y(1)-Y(2))
C            1 /(X(1)-X(2)) - (Y(2)-Y(3))/(X(2)-X(3)))
C            XP = 0.5D0*(AUX(1)/(Y(1)-Y(2)) - AUX(2)/(Y(2)-Y(3)))/((X(1)-X(2))
C            1 /(Y(1)-Y(2)) - (X(2)-X(3))/(Y(2)-Y(3)))
C          DO 30 I = 1,NT
C            TRANS(I) = TRANS(I)/SV
30 CONTINUE
C          RETURN
C          END

C          SUBROUTINE SETUP(X,Y,Z,XP,YP,ZP,SV,N,NT,NV,NT3,NMAX,A,S,ICOL,IROW,
C          1 VALUE)
C          IMPLICIT REAL*8(A-H,O-Z)

C          THIS MATRIX SETS UP THE A AND S MATRICES NEEDED FOR THE BLOCK
C          BY BLOCK FORMATION OF THE NORMAL MATRIX.
C          INPUT:  X(I), I = 1,NT X COORDINATES FOR THE TRANSPONDERS
C                  Y(I), I = 1,NT Y      "      "      "      "
C                  Z(I), I = 1,NT Z      "      "      "      "
C                  XP,YP,ZP, APPROXIMATE COORDINATES FOR PEGASUS.
C                  SV    = VELOCITY OF SOUND THROUGH WATER.
C                  N      = NTH POSITION OF PEGASUS.
C                  NT, NV, NT3 SAME AS IN THE MAIN PROGRAM.
C                  NMAX   = ACTUAL # OF PEGASUS POSITIONS IN THIS DROP
C          OUTPUT: A = THE PORTION OF THE A MATRIX CORRESPONDING TO THE
C                   NTH PEGASUS POSITION.
C                   S = AS ABOVE, BUT THE OUTER BAND OF THE A MATRIX.
C                   VALUE = THE NON ZERO # IN THE A MATRIX CORRESPONDING
C                           TO THE COLUMN # HELD IN ICOL AND THE ROW # HELD
C                           IN IROW.
C
C          DIMENSION X(NT), Y(NT), Z(NT), A(NT,3), S(NT,NT3), VALUE(NV)
C          INTEGER*2 ICOL(NV), IROW(NV)
C
C          NROW = (N-1)*NT
C          NCOL = (N-1)*3
C          NCOL1 = NMAX*3
C          ICOUNT = (N-1)*6*NT
C          DO 20 I = 1,NT
C            J = 1
C            K = (I-1)*3 + 1
C            DIST = DSQRT((XP - X(I))**2 + (YP - Y(I))**2 + (ZP - Z(I))**2)

```



```

A(I,J) = (XP - X(I))/(DIST*SV)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = A(I,J)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL + J
A(I,J+1) = (YP - Y(I))/(DIST*SV)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = A(I,J+1)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL + J+1
A(I,J+2) = (ZP - Z(I))/(DIST*SV)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = A(I,J+2)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL + J+2
S(I,K) = -A(I,J)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = S(I,K)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL1 + K
S(I,K+1) = -A(I,J+1)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = S(I,K+1)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL1 + K+1
S(I,K+2) = -A(I,J+2)
ICOUNT = ICOUNT + 1
VALUE(ICOUNT) = S(I,K+2)
IROW(ICOUNT) = NROW + I
ICOL(ICOUNT) = NCOL1 + K+2
20 CONTINUE
WRITE(2,100)
100 FORMAT('1',///,15X,'A MATRIX AND THEN THE S MATRIX, BLOCK BY',
1 ' BLOCK')
WRITE(2,101)((A(I,J),J=1,3),I=1,4)
101 FORMAT(' ', T2,4(2X,3(F10.8,3X),/))
WRITE(2,102)((S(I,J),J=1,12),I=1,4)
102 FORMAT(' ', T2,4(12(F10.7,1X),/))
RETURN
END

```

```

SUBROUTINE NORM(A,S,P,L,N,NT,NT3,NN,NP,NMAX,ATP,ATPA,ATPS,ATPL,
1 STP,STPS,STPL,ANORM,XHAT)
IMPLICIT REAL*8(A-H,O-Z)

```

```

C
C THIS SUBROUTINE FORMS THE NORMAL EQUATIONS, BLOCK BY BLOCK
C INPUT: A MATRIX FROM SUBROUTINE SETUP
C S " " " "
C P THE WEIGHT MATRIX CORRESPONDING TO THIS BLOCK
C L THE L MATRIX " " "
C N, NT, NT3, AS FOR SUBROUTINE SETUP
C NN, NP, NMAX, AS FOR MAIN PROGRAM
C OUTPUT: A NUMBER OF AUXILLARY MATRICES USED IN THE MATRIX
C MULTIPLICATIONS, IE., ATP,ATPA,ATPS,ATPL,STP,STPS,
C AND STPL.
C MATRICES SPS AND SPL WHICH COME IN VIA THE COMMON

```

```

C          BLOCK ARE UPDATED.
C          ANORM - THE NORMAL MATRIX (IN ARRAY FORM)
C          XHAT  - THE ATPL VECTOR
C
      DIMENSION A(NT,3), S(NT,NT3),P(NT,NT),ATP(NT,3), ATPA(3,3),
1  ATPS(3,NT3), ATPL(3,1), STP(NT3,NT), STPS(NT3,NT3),STPL(NT3,1),
2  ANORM(NN),XHAT(NP)
      DOUBLE PRECISION L(NT,1)
      COMMON SPS(12,12), SPL(12,1)
C
C  PERFORM THE MATRIX MULTIPLICATIONS
C
      CALL ATB(A,P,ATP,NT,3,NT)
      CALL AB(ATP,A,ATPA,3,NT,3)
      CALL AB(ATP,S,ATPS,3,NT,NT3)
      CALL AB(ATP,L,ATPL,3,NT,1)
      CALL ATB(S,P,STP,NT,NT3,NT)
      CALL AB(STP,S,STPS,NT3,NT,NT3)
      CALL AB(STP,L,STPL,NT3,NT,1)
      WRITE(2,100)((ATPA(I,J),J=1,3),ATPL(I,1),I=1,3)
100  FORMAT(' ',/,15X, 'ATPA BLOCK',10X,'ATPL VECTOR',//,
1  3(2X,3(D10.3,2X), 4X,D10.3,/))
      WRITE(2,110)((STPS(I,J),J=1,12),I=1,12)
110  FORMAT(' ',/,15X, 'STPS BLOCK',//,
1  T2,12(12(D10.3,1X),/))
C
C  PLACE ATPA AND ATPL IN THEIR APPROPRIATE POSITION IN EITHER THE
C  NORMAL MATRIX OR THE COLUMN VECTOR
C
      NCOL = (N-1)*3 + 1
      I1 = NCOL*(NCOL + 1)/2
      ANORM(I1) = ATPA(1,1)
      I11 = I1 + NCOL
      ANORM(I11) = ATPA(1,2)
      ANORM(I11 + 1) = ATPA(2,2)
      I111 = I11 + 1 + NCOL
      ANORM(I111) = ATPA(1,3)
      ANORM(I111 + 1) = ATPA(2,3)
      ANORM(I111 + 2) = ATPA(3,3)
C
      DO 10 I = 1,NT3
      INT = (NMAX*3) + I
      J = INT*(INT - 1)/2 + 1 + (N-1)*3
      ANORM(J) = ATPS(1,I)
      ANORM(J+1) = ATPS(2,I)
      ANORM(J+2) = ATPS(3,I)
10  CONTINUE
C
      DO 20 I = 1,3
      XHAT(NCOL+I-1) =ATPL(I,1)
20  CONTINUE
C
C  UPDATE SPS AND SPL
C
      DO 30 I = 1,NT3

```

```

      SPL(I,1) = SPL(I,1) + STPL(I,1)
      DO 30 J = 1,NT3
      SPS(I,J) = SPS(I,J) + STPS(I,J)
30 CONTINUE
      RETURN
      END

```

```

      SUBROUTINE ATB(A,B,R,L,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
C
C   THIS SUBROUTINE COMPUTES THE MATRIX PRODUCT A'B
C   INPUT:  MATRIX A (L X M)
C           MATRIX B (L X N)
C   OUTPUT: MATRIX R (M X N)
C
      DIMENSION A(L,M),B(L,N),R(M,N)
      DO 10 I = 1,M
      DO 10 J = 1,N
      R(I,J) = 0.0D0
      DO 5 K = 1,L
      R(I,J) = R(I,J) + A(K,I)*B(K,J)
      5 CONTINUE
      10 CONTINUE
      RETURN
      END

```

```

      SUBROUTINE AB(A,B,R,L,M,N)
      IMPLICIT REAL*8(A-H,O-Z)
C
C   FORM THE MATRIX PRODUCT R = AB.
C   THE MATRICES A AND B ARE RETURNED UNCHANGED
C
      DIMENSION A(L,M),B(M,N),R(L,N)
      DO 5 I = 1,L
      DO 5 J = 1,N
      R(I,J) = 0.0D0
      DO 5 K = 1,M
      R(I,J) = R(I,J) + A(I,K)*B(K,J)
      5 CONTINUE
      RETURN
      END

```

```

      SUBROUTINE RESID(VALUE,ICOL,IROW,BIGP,L1,XHAT,NMAX,NT,NV,NP,NOBS,
1 P,VTP,V,SIG,V1,SIGO)
      IMPLICIT REAL*8(A-H,O-Z)
C
C   COMPUTES THE RESIDUALS ON ALL THE TRANSPONDER TIME DELAY
C   OBSERVATIONS AND THE A POSTERIORI VARIANCE OF UNIT WEIGHT.
C   INPUT: VALUE, ICOL,IROW, - MATRICES DERIVED IN SUBROUTINE SETUP
C           BIGP - THE FULL WEIGHT MATRIX FOR ALL OBSERVATIONS
C           L1 - THE VECTOR OF "COMPUTED-OBSERVED" OBSERVATIONS
C           XHAT - THE LEAST SQUARES SOLUTION VECTOR
C           NMAX,NT,NV,NP,NOBS - AS IN THE MAIN PROGRAM
C   AUXILLARY MATRICES : P, VTP, V,SIG
C   OUTPUT: V1 - THE VECTOR OF RESIDUALS
C           SIGO - THE A POSTERIORI VARIANCE OF UNIT WEIGHT

```

```

C      DIMENSION VALUE(NV),XHAT(NP),V1(NOBS),BIGP(NT,NOBS),P(NT,NT),
1 VTP(1,NT),SIG(1,1),V(NT,1)
      DOUBLE PRECISION L1(NOBS)
      INTEGER*2 ICOL(NV), IROW(NV)
C
      DO 10 I = 1,NOBS
        V1(I) = 0.0D0
10 CONTINUE
      IC = 1
15 DO 20 I = 1,NV
        IF(IROW(I).NE.IC) GO TO 18
16        V1(IC) = V1(IC) + VALUE(I)*XHAT(ICOL(I))
          GO TO 20
18        V1(IC) = V1(IC) + L1(IC)
          IC = IC + 1
          IF(IC.LE.NOBS) GO TO 16
20 CONTINUE
      V1(NOBS) = V1(NOBS) + L1(NOBS)
C
      SIGO = 0.0D0
      IC = 0
      DO 40 I = 1,NMAX
        DO 30 J = 1,NT
          V(J,1) = V1(IC*NT + J)
          DO 30 K = 1,NT
            P(J,K) = BIGP(J,IC*NT + K)
30        CONTINUE
          CALL ATB(V,P,VTP,NT,1,NT)
          CALL AB(VTP,V,SIG,1,NT,1)
          SIGO = SIGO + SIG(1,1)
          IC = IC + 1
40 CONTINUE
      SIGO = SIGO/FLOAT(NOBS-NMAX*3)
      RETURN
      END

      SUBROUTINE P2L2(P2,LB2,XO,ANORM,L2,XHAT,NP,NN)
      IMPLICIT REAL*8(A-H,O-Z)
C
C      THIS SUBROUTINE COMPUTES THE L2 MATRIX AND ADDS IT TO THE COLUMN
C      VECTOR. IN ADDITION IT INCREMENTS THE DIAGONAL ELEMENTS OF THE
C      NORMAL MATRIX TO INCLUDE THE INFLUENCE OF P2
C
      DIMENSION P2(NP), XO(NP),ANORM(NN),XHAT(NP,1)
      DOUBLE PRECISION L2(NP), LB2(NP)
C
      DO 10 I = 1,NP
        L2(I) = XO(I) - LB2(I)
        INT = I*(I+1)/2
        ANORM(INT) = ANORM(INT) + P2(I)
        P2L = P2(I)*L2(I)
        XHAT(I,1) = XHAT(I,1) + P2L

```

```

10 CONTINUE
   RETURN
   END

C
   SUBROUTINE MAT(C,NMAX)
C  *NOT PART OF J HANNAH'S ORIGINAL PROGRAM.  ADDED BY M. HASKELL.
C  THIS PROGRAM READS APPROPRIATE VALUES FROM THE VARIANCE-COVARIANCE
C  MATRIX FILE (ANORM) AND PLACES THEM IN THE MATRICES NEEDED TO COM-
C  PUTE VARIANCES OF THE CURRENT VELOCITIES.
      IMPLICIT REAL*8 (A-H,O-Z)
      DOUBLE PRECISION C(1)
C      DIMENSION C(444153)
      I(K) = K*(K+1)/2
      K = 1
      ROW = K
      WRITE (2,*)C(I(K)), C(I(K+1)-1), C(I(K+2)-2)
      WRITE (2,*)C(I(K+1)-1), C(I(K+1)), C(I(K+2)-1)
      WRITE (2,*)C(I(K+2)-2), C(I(K+2)-1), C(I(K+2))
      DO 15 J = 1,NMAX+1
        WRITE (2,*)C(I(K+3)-3), C(I(K+4)-4), C(I(K+5)-5)
        WRITE (2,*)C(I(K+3)-2), C(I(K+4)-3), C(I(K+5)-4)
        WRITE (2,*)C(I(K+3)-1), C(I(K+4)-2), C(I(K+5)-3)
C
        WRITE (2,*)C(I(K+3)), C(I(K+4)-1), C(I(K+5)-2)
        WRITE (2,*)C(I(K+4)-1), C(I(K+4)), C(I(K+5)-1)
        WRITE (2,*)C(I(K+5)-2), C(I(K+5)-1), C(I(K+5))
        K = K+3
15 CONTINUE
C
      WRITE (2,*)C(I(K+3)-6), C(I(K+4)-7), C(I(K+5)-8)
      WRITE (2,*)C(I(K+3)-5), C(I(K+4)-6), C(I(K+5)-7)
      WRITE (2,*)C(I(K+3)-4), C(I(K+4)-5), C(I(K+5)-6)
C
      WRITE (2,*)C(I(K+3)-3), C(I(K+4)-4), C(I(K+5)-5)
      WRITE (2,*)C(I(K+3)-2), C(I(K+4)-3), C(I(K+5)-4)
      WRITE (2,*)C(I(K+3)-1), C(I(K+4)-2), C(I(K+5)-3)
C
      WRITE (2,*)C(I(K+3)), C(I(K+4)-1), C(I(K+5)-2)
      WRITE (2,*)C(I(K+4)-1), C(I(K+4)), C(I(K+5)-1)
      WRITE (2,*)C(I(K+5)-2), C(I(K+5)-1), C(I(K+5))
C
      K = K+3
      WRITE (2,*)C(I(K+3)-9), C(I(K+4)-10), C(I(K+5)-11)
      WRITE (2,*)C(I(K+3)-8), C(I(K+4)-9), C(I(K+5)-10)
      WRITE (2,*)C(I(K+3)-7), C(I(K+4)-8), C(I(K+5)-9)
C
      WRITE (2,*)C(I(K+3)-6), C(I(K+4)-7), C(I(K+5)-8)
      WRITE (2,*)C(I(K+3)-5), C(I(K+4)-6), C(I(K+5)-7)
      WRITE (2,*)C(I(K+3)-4), C(I(K+4)-5), C(I(K+5)-6)
C
      WRITE (2,*)C(I(K+3)-3), C(I(K+4)-4), C(I(K+5)-5)
      WRITE (2,*)C(I(K+3)-2), C(I(K+4)-3), C(I(K+5)-4)
      WRITE (2,*)C(I(K+3)-1), C(I(K+4)-2), C(I(K+5)-3)

```

```

C      WRITE (2,*)C(I(K+3)), C(I(K+4)-1), C(I(K+5)-2)
      WRITE (2,*)C(I(K+4)-1), C(I(K+4)), C(I(K+5)-1)
      WRITE (2,*)C(I(K+5)-2), C(I(K+5)-1), C(I(K+5))
C
C      K = 36
C      DO 15 J = 1,6
C          WRITE (2,*)C(I(K+3)-3), C(I(K+4)-4), C(I(K+5)-5)
C          WRITE (2,*)C(I(K+3)-2), C(I(K+4)-3), C(I(K+5)-4)
C          WRITE (2,*)C(I(K+3)-1), C(I(K+4)-2), C(I(K+5)-3)
C
C          WRITE (2,*)C(I(K+3)), C(I(K+4)-1), C(I(K+5)-2)
C          WRITE (2,*)C(I(K+4)-1), C(I(K+4)), C(I(K+5)-1)
C          WRITE (2,*)C(I(K+5)-2), C(I(K+5)-1), C(I(K+5))
C      K = K+36
C 15  CONTINUE
      RETURN
      END

```

APPENDIX B

CURRENT VELOCITY VARIANCE-COVARIANCE PROGRAM

This program computes the variance-covariance matrix of the current velocities by a procedure described in Chapter VI, Section D. It uses data from the Pegasus position variance-covariance matrix produced by the Fortran program Pegasus as described in Appendix A.

```

C   THIS PROGRAM USES DATA FROM THE VARIANCE-COVARIANCE MATRIX PRODUCED
C   BY THE PEGASUS FORTRAN PROGRAM.  IT COMPUTES THE VARIANCE-COVARI-
C   ANCE MATRIX OF THE CURRENT VELOCITIES.
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL SP1(3,3), SP12(3,3), SP2(3,3), V(3,3), SU, SV, SW
C
      N = 1
      DO 100 I = 1,3
        READ(1,*)(SP1(I,J),J = 1,3)
        WRITE(2,120)(SP1(I,J),J = 1,3)
C      120   FORMAT(' ',3X,F10.4,3X,F10.4,3X,F10.4)
      100 CONTINUE
C
      10   DO 200 I = 1,3
        READ(1,*,END=45)(SP12(I,J),J = 1,3)
        WRITE(2,120)(SP12(I,J),J = 1,3)
C      200 CONTINUE
      DO 300 I = 1,3
        READ(1,*)(SP2(I,J),J = 1,3)
        WRITE(2,120)(SP2(I,J),J = 1,3)
C      300 CONTINUE
C
      CC   WRITE(2,15)N
      15   FORMAT(' ',/,5X,'VELOCITY VARIANCE-COVARIANCE MATRIX FOR ',
        1' POSITION ',I3,/)
      DO 400 I = 1,3
        DO 350 J = 1,3
          V(I,J) = ((SP1(I,J) + SP2(I,J) - SP12(I,J) - SP12(J,I)) / (16**2))
        350 CONTINUE
C      WRITE(2,120)(V(I,J),J = 1,3)
      400 CONTINUE
      SU = SQRT(V(1,1))
      SV = SQRT(V(2,2))
      SW = SQRT(V(3,3))
      CC   WRITE(2,130)SU, SV, SW
      CC130 FORMAT(' ',/,3X,'SIGMA U =',F10.4,3X,'SIGMA V =',F10.4,3X,
      CC   1'SIGMA W =',F10.4/)
      WRITE(2,600)N,SU,SV,SW
      600 FORMAT(' ',10X,'POS ',I3,3X,F7.4,3X,F7.4,3X,F7.4)
      DO 450 I = 1,3
        DO 460 J = 1,3
          SP1(I,J) = SP2(I,J)
        460 CONTINUE
C      WRITE(2,140)(SP1(I,J),J=1,3)
C      140   FORMAT(' ',3X,F10.4,3X,F10.4,3X,F10.4)
      450 CONTINUE
      N = N + 1
      GO TO 10
      45   CONTINUE
      STOP
      END

```


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